# Financial Crises, Debt Maturity, and Capital Controls 

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November 27, 2019


#### Abstract

This paper studies debt portfolio choice and optimal capital control policy in an open economy with financial frictions. I construct a new measure of capital control changes and document two novel stylized facts during financial crises: (i) capital inflow controls are tightened, and (ii) short-term inflow controls are tightened more than long-term inflow controls. Motivated by these empirical findings, I extend the model of international borrowing with collateral constraint to allow for multiple debt maturities. As in the single-maturity version of the model, the equilibrium exhibits overborrowing because, due to a pecuniary externality, private agents undervalue the cost of financial liabilities that demand repayment in future constrained states. The key insight of the multiple-maturity model is that overborrowing in short-term debt is especially severe because the repayment of short-term liabilities is larger than that of long-term liabilities in future constrained states, resulting in greater cost undervaluation of short-term financial obligations. To counteract these inefficiencies, the model justifies a set of maturity-dependent capital controls. In line with the data, the model predicts a tightening of capital controls tilted toward short maturities during financial crises. When calibrated to Argentine data, the model reproduces the observed dynamics of debt portfolios, and the short-term targeting of capital controls during crises. The optimal capital-control policy reduces the frequency of crises by half and generates sizable welfare improvements.


Key Words: debt portfolio choice, optimal capital control policy, collateral constraint, financial crisis JEL Codes: F41, F42, F44

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## 1 Introduction

Episodes of financial crisis are often associated with two patterns that have been extensively documented: large movements in capital flows ${ }^{1}$ and drastic shift to shorter-maturity flows. ${ }^{2}$ The first pattern, namely the surges and stops in capital flows, has resulted in a strong push to revamp capital control as an essential form of regulation. ${ }^{3}$ Regarding the second pattern about short-term flows, the emerging view emphasizes that short-term liabilities render an economy particularly vulnerable to rollover crises or self-fulfilling liquidity crises and that long-term debt should be favored. ${ }^{4}$ However, if the risk of short-term debt is correctly perceived and incorporated in optimal decisions, then the large proportion of short-term debt means that the gains from increased short-term debt today exceed the expected costs of financial distress in the future. Therefore, the questions remain: is short-term liability accumulation inefficient ex ante? Under what conditions does private agents' optimization lead to inefficient debt portfolio choice compared with the social optimal? What should be the design of optimal regulating policies to counteract the inefficiencies?

This paper aims to address these questions from both empirical and theoretical perspectives. I argue that with financial frictions, (i) the debt portfolio in the competitive economy is inefficient in terms of overborrowing and excess short-term debt issuing, and (ii) these inefficiencies can be eliminated through the adoption of a set of state-contingent and maturity-dependent capital inflow controls. Empirically, I examine the behavior of capital controls in practice. To this end, I propose a new measure to closely trace the changes in the stringency of capital controls. Using this new measure, I analyze the cyclicality and maturity-dependency of capital controls. Theoretically, I develop a small open economy model where external borrowing is available in multiple maturities yet subject to collateral constraint and risk-averse international creditors. I use the model to analyze the mechanism inducing inefficiency, to derive and verify optimal capital control policy, and to evaluate welfare improvement generated by the optimal policy.

The analysis begins with documenting two novel stylized facts about the behavior of capital inflow controls during financial crisis episodes. To do so, I first construct a new measure of capital control stringency based on the changes in capital control policies. This measure is built on a thorough

[^1]analysis of the narratives about the capital account regulation changes, ${ }^{5}$ and it distinguishes whether the policy change targets inflow or not, whether it is tightening or easing, and whether it targets short-term inflow, long-term inflow, or neither. Utilizing the rich time variation captured by the new index compared to the existing measures, I show that in financial crisis episodes, (i) inflow controls are tightened: $12.5 \%$ of crisis years display inflow tightening, which triples the non-crisis counterpart; (ii) short-term inflow is tightened to a larger extent: in general, short-term inflow tightening is employed much more often than on the long-term, and in crisis, the occurrence of short-term inflow tightening is twice that of long-term. The first fact is consistent with Na et al. (2014) who study sovereign debt crisis episodes, and the second one is new to the literature. I complement this evidence by examining the reserve requirements on foreign currency demand deposit (short-term) and term deposit (long-term), and show that the dynamics in reserve requirements also align with the cyclicality and short-term inflow targeting: the demand deposit's reserve requirement is significantly higher than that of term deposit, and it increases by $8 \%$ in crisis exceeding the $3 \%$ mild rise in term deposit.

To rationalize these stylized patterns and inspect the underlying mechanism, I build a model that embeds multiple debt maturities and risk-averse international creditors in a standard collateral constraint model à la Korinek (2017) and Bianchi (2011). As in the single-maturity version of the model, the equilibrium exhibits overborrowing because, due to a pecuniary externality, private agents undervalue the cost of financial liabilities that demands repayment in future constrained states. The key insight of the multiple-maturity model is that overborrowing in short-term debt is especially severe because the repayment of short-term liabilities is larger than that of long-term liabilities in future constrained states. Therefore, pecuniary externality leads to two sorts of inefficiency in private agents' debt portfolio decision: (i) overborrowing regardless of debt maturity, which is in line with the literature such as Korinek (2017) and Bianchi (2011), and more importantly (ii) excess short-term debt issuing, which is new to the literature.

How does pecuniary externality play a role in shaping debt portfolio? In the model, optimal debt portfolio is determined by two sets of trade-offs: inter-temporal trade-off and the intra-temporal trade-off. On the inter-temporal perspective, private agents trade off between the current benefit of borrowing and the associated future repayment cost. However, the repayment cost is undervalued in the future states where collateral constraint binds. This undervaluation arises because by taking collateral price as given, private agents evaluate the cost of repayment as the utility loss from the

[^2]direct one-to-one consumption decrease. However, they neglect that, with collateral constraint binding, debt repayment triggers financial amplification: consumption decrease lowers collateral price and value, limits borrowing capacity, and further constrains consumption. This contractionary spiral magnifies the future repayment cost, yet it is not internalized by private agents.

On the intra-temporal perspective, the key difference between the short- and long-term debt is the amount of required repayment. Given current short-term borrowing rate, its demanded repayment is independent with next period's states. On the contrary, the price of long-term debt decreases in response to interest rate hike, which is often associated with future crisis states, exhibiting insurance against adverse shocks. Therefore, long-term debt's repayment cost is effectively low in future collateral constrained states. Given the same undervaluation per unit repayment, the low repayment level of long-term debt leads to milder cost undervaluation than that of the short-term, resulted in excess short-term debt issuing.

In order to counteract these inefficiencies stemming from pecuniary externality, I prove that the social optimal debt portfolio can be decentralized by a set of state-contingent and maturitydependent capital controls, and I derive a close-form solution for the capital control taxes. To shed further light on the design of capital control policy, I analyze two aspects: cyclicality and maturity-dependence. I first focus on the cyclicality, which is equivalent to the cyclicality of the level of undervaluation in future repayment cost. The magnitude of this undervaluation is determined by two factors: the probability of future crisis and the magnitude of financial amplification effect. Due to the persistence of shocks, crisis in current period indicates a high probability of future crisis, and the low consumption in crisis leads to a large financial amplification effect. Therefore, pecuniary externality in crisis exceeds that in normal time, which directly yields capital controls tightening for both short- and long-term inflows in crisis.

Another important feature of the optimal capital control policy is the short-term inflow targeting, which hinges on the relative magnitude of repayment cost undervaluation between short-term debt and long-term debt. Essentially, this level of relative undervaluation can be decomposed into the product of (i) the undervaluation of future cost per unit of repayment (ii) the magnitude of repayment difference between short-term and long-term debt. The former peaks in crisis, following the same logic as in the analysis of cyclicality. The latter is the new ingredient, which is governed by the term premium. As is supported by the data, term premium surges in crisis. Therefore, compared with the long-term debt, the repayment cost of short-term debt is more undervalued, and most severely undervalued in crisis, which requires tighter short-term inflow control in general, and
especially tighter in crisis.
When calibrated to Argentine data, the model can reproduce external borrowing collapse, maturity shrinking, and the behavior of the key aggregate variables in crisis. The model also successfully generates the overborrowing and excess short-term debt of the competitive economy compared with the social optimum. In terms of the optimal capital control policies, the model derives a set of capital inflow taxes that hike in crisis and target short-term inflow, consistent with the empirical findings.

The quantitative analysis also shows significant welfare improvements by the optimal capital controls. Crisis frequency drops by half, and the severity of crisis is substantially alleviated, for instance, the magnitude of tradable consumption decrease reduces by $17 \%$, the real exchange rate depreciation is $24 \%$ smaller. On average, the improvement in life-time utility is equivalent to $0.59 \%$ consumption increase. The welfare analysis also demonstrates the importance of distinguishing maturity in capital controls because the maturity-independent capital controls can only achieve half of the welfare improvement generated by the maturity-dependent counterpart. Comparing the welfare effects by short-term inflow control and that of the long-term, the analysis shows that the welfare improvement from exclusive long-term inflow control produces only $10 \%$ of that from exclusive short-term inflow control. This result indicates that if policymakers are constrained in policy tools then short-term inflow control should be set as the priority.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the empirical facts about external borrowing and capital control policies in financial crisis episodes. Section 4 builds a dynamic small open economy model including external borrowing under collateral constraint with different maturities and risk-averse international creditors. Section 5 discusses the main mechanism relating debt portfolio choice and optimal capital control policy, and it also presents the model calibration, the quantitative results, and the optimal policy evaluation. Section 6 concludes and discusses possible future extensions.

## 2 Related Literature

This paper contributes towards four main strands of literature. First, this paper adds to the growing quantitative studies of pecuniary externality due to collateral constraints and the remedies. Seminal papers of Korinek (2017), Jeanne and Korinek (2010), and Bianchi (2011), show the presence of market price in collateral constraint generates pecuniary externality which calls for capital controls.

Recent papers have extended the model in different directions, for instance, production economy as Benigno et al. (2013), alternative policy instruments as Benigno et al. (2016), time consistent policy as Bianchi and Mendoza (2013) and Devereux, Young and Yu (2015), cyclicality of capital control as Schmitt-Grohé and Uribe (2017). However, the existing works have focused exclusively on the interaction between pecuniary externality and short-term debt. ${ }^{6}$ This paper builds on these studies by introducing endogenous debt maturity structure and showing that pecuniary externality leads to inefficiencies in not only quantity of debt but also maturity structure of debt. These dual effects of pecuniary externality give rise an extra layer in optimal policy design that both state-contingency and maturity-dependence are necessary to restore social optimal equilibrium.

Second, this paper is related to the literature on the optimal maturity structure of debt. There has been a growing strand of works studying the maturity structure of sovereign debt, including Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Broner, Lorenzoni and Schmukler (2013), and Aguiar and Amador (2013), rationalizing the relationship between maturity structure, borrowing cost, and default decision. Different from their focus on the financial friction resulted from no repayment commitment, I study another form of financial friction as the limit of borrowing capacity based on the pledged collateral. Despite the difference in modeling financial friction, this paper shares a similar logic with the defaultable debt literature in the trade-off underlying optimal maturity choice, such as the insurance benefit of long-term debt and the cost benefit of short-term debt. Moreover, I focus on the external borrowing from the private sector and the ineffcient in the competitive equilibrium compared with social optimum, and this new angle provides the room for studying policy interventions. Within the defaultable debt literature, there have been works discussing the disavantage of short-term debt in terms of rollover risk and self-fulling liquidity risk, for instance, Cole and Kehoe (2000), Alfaro and Kanczuk (2009), and Jeanne (2009). However, this paper differs from them in analyzing the ex ante efficiency of debt portfolio choice and provide a close form illustration of the optimal policy. There is also a large literature in corporate finance on the optimal maturity structure of debt. Recent developments include short-term debt and rollover risk, including Brunnermeier and Yogo (2009), He and Xiong (2012), Farhi and Tirole (2012), and Brunnermeier and Oehmke (2013). Instead of these static models, this paper examines the optimal maturity structure and its cyclical features in a dynamic framework where financial crises endogenously rise.

[^3]Third, the empirical analysis on capital control policies during financial crisis episodes is related to the recent literature on capital control policy implementation in practice. Fernández, Rebucci and Uribe (2015) provide a comprehensive analysis showing that capital control is acyclical. Focusing on the sovereign default crises, Na et al. (2014) find that capital controls are significantly tightened. This paper contributes the literature by analyzing the dynamics of capital control policies in a new set of important economy episodes: financial crises. Similar to Na et al. (2014), I also find capital controls are more restricted in the bust of crisis episodes. Combining with the existing evidence, this implies there might be non-linear relationship between capital control policies and business cycle in a way that the policy is likely to respond to only severe shocks in economy. As a result, capital control policy seems not responding to business cycle in general but only to significant economy fluctuations.

Fourth, the newly constructed index on the capital control changes complements the existing measures of capital controls. The widely used measures in the literature include Chinn and Ito (2008), Schindler (2009), and Fernández et al. (2016). However, all of them are based on the capital control levels, i.e. whether there are controls for a certain type of capital trasaction. Instead of level which is the extensive margin of capital controls, the newly constructed measure is based on the changes in capital control policies which capture the intensive margin. There have been works looking into capital control changes for different countries and different time periods, such as Pasricha et al. (2015) and Forbes, Fratzscher and Straub (2015). However, a new perspective brought by this measure is the distinction between short-term flow and long-term flow capital controls. Despite of the policy attention on managing short-term capital flows, there lacks systematic empirical analysis, and this measure intends to shed some light on the potential maturity-adjust mandate in capital control policy implementations in practice.

## 3 Stylized Facts on External Borrowing and Capital Controls in Financial Crises

This section examines the patterns of private external borrowing and capital control policy implementation in financial crisis episodes. Specifically, this analysis focuses on the amount of private external borrowing, the maturity choice, and the capital inflow regulations around financial crisis episodes. The external borrowing data shows that the amount of private external borrowing collapses during financial crises and the private debt portfolio significantly tilts towards short-term borrowing,
consistent with the literature. More importantly, there emerge two novel stylized facts in capital inflow control. First, capital inflow controls are tightened in crisis compared with pre- and post-crisis. Second, short-term capital inflow is tightened by a greater extent than the long-term inflow.

## A. External Borrowing and Maturity Structure

Utilizing the International Debt Statistics dataset (1970-2016) from the World Bank, I calculate external borrowing as the change in external debt stock in private sector and measure maturity structure by the share of private sector short-term debt in total private sector debt. For the interest of financial crises, I focus on the 139 financial crisis windows (114 countries, 1970-2011) identified by (Laeven and Valencia, 2013) and evaluate the changes in the quantity and the maturity composition of private external borrowing across crisis windows.

Figure 1 displays the average of external borrowing and the share of short-term debt, both normalized as relative to the level three-year prior to crisis, among all the seven-year financial crisis windows (three-year lags and three-year leads). It can be drawn that upon the arrival of financial crisis, private external borrowing remarkably moves away from the pre- and post-crisis levels. On the amount of external borrowing, the total debt issuance falls by $60 \%$ compared to the pre-crisis level. On the maturity composition of external debt portfolio, the share of short-term debt increases by more than $30 \%$ from the normal time's level. Figure A. 1 and Figure A. 2 provide further evidence

Figure 1: External Financing in Financial Crises



Note: This figure plots the dynamics of external borrowing and short-term external debt share in a seven-year window of 98 financial crises. The magnitude is normalized as relative to 3 -year prior to the starting year of crisis.
about the collapse in external borrowing and increase in short-term borrowing in typical financial
crises such as the Asian financial crises in late 1990s and the Latin American 1990s crises. The results demonstrate that the changes are in much greater magnitude than the usual business cycle fluctuations. For instance, in Indonesia, Malaysia, and Thailand during 1997/1998 financial crisis, the external borrowing plummted, with a drop of almost $90 \%$ from pre-crisis level. Moreover, in all the crises, the share of short-term debt peaks in the crisis windows, with the increase at least $11 \%$ as compared to the pre-crisis portfolio. These results, although for private sector in financial crises, are consistent with the facts have been documented for sovereign external borrowing, for instance, Arellano and Ramanarayanan (2012) and Broner, Lorenzoni and Schmukler (2013).

## B. Changes in Capital Controls in Financial Crises

Measuring Capital Controls - Having discussed the external borrowing decisions of private sector in financial crises, I now consider the government's policy responses regarding these drastic changes in external financing. To do so, I construct a new measurement on the changes in capital control policies based on the detailed policy narratives. Focusing on the same financial crisis episodes as the last section, this new dataset contains a set of capital control measures on a five-year window, spanning from two years before the crisis to two years after the start of the crisis.

To quantify the changes of capital control, I employ the information from the "Changes" section from the Annual Report on Exchange Arrangements and Exchange Restrictions of IMF (AREAER). In each year, IMF publishes the AREAER on member country's policy regarding various sorts of exchange rate management, payment and proceeds arrangement, capital transaction and other transactions. For each type of transaction, the report documents whether there is a restriction or not and what specific requirements are imposed. In every country's profile, after the detailed description of arrangements and restrictions by transaction type, there is a "Changes" section detailing all the policy changes happened in the past year. It is constituted by a list of policy narratives categorized by the general types of transactions in a chronological manner. For instance, on November 28th 2008. Iceland initiated a new regulation on credit transaction within capital transaction category: "Borrowing from nonresidents is allowed only for loans with a maturity of at least one year and in an amount not exceeding ISK 10 million a person a calendar year." To quantify the information contained in these narratives, I inspect each statement of policy change from three dimensions: the direction of the flow (into the country or out from the country), the regulating intention of the policy (tightening or easing), and the target maturity (short-term, or long-term, or maturity independent). For instance, the previously mentioned new policy carried out in Iceland on November 28th 2008
is labeled as both "short-term inflow tightening". Combining all the information extracted from policy narratives, I collect 789 capital control changes policy statements, and I categorize them into size groups: short-term inflow tightening, short-term inflow easing, long-term inflow tightening, long-term inflow easing, maturity-independent tightening, and maturity-independent easing. ${ }^{7}$ In the end, I construct the capital control change measures as the numbers of inflow tightening/easing policies for each year in the financial crisis window.

This newly constructed capital control measure aligns with the body of literature quantifying capital controls, however, it has two unique advantages compared to the existing capital control indexes. The first advantage is that it provides a way to investigate the changes in capital control at intensive margin besides the existing extensive margin measures. In other words, it helps us to understand how much less/more stricter the controls become in the cases other than a $100 \%$ liberalization or a complete close-up. For instance, in the country report of Ukraine in AREAER (2009), there is capital control change: "Ukraine 2008: reserve requirement on deposits and loans in foreign currency from nonresidents is increased from $4 \%$ to $20 \%$ for a term not exceeding 183 calendar days". The new measure logs this as a "short-term capital inflow control tightening", however, this change does not generate time variation in the existing measures on capital control levels because for both 2008 and 2009 credit transaction is documented as "constrained" (by certain level of reserve requirements).

In order to evaluate the effectiveness of this new index in capturing the dynamics in capital control policy, I compare it to the widely used existing measurements. As discussed before, besides the approach applied here, there are two other ways in the literature aiming to closely trace the fine development in capital controls. The first one is to assign values to different restrictions to reflect their different intensities, applied in Quinn, Schindler and Toyoda (2011). The second method is to look into detailed types of capital transactions and aggregate up the overall capital control measure from multiple transaction-specific measuress, applied in Schindler (2009) and Fernández et al. (2016). The two approaches both succeed in restoring more variations than an overall measure at the aggregate level.

Table 1 presents the comparison among three capital control measures in terms of the number of capital control changes captured in the overlapping years and countries. The top panel of Table 1 reports the comparisons between the new measure and the Quinn index. Since the Quinn index does not distinguish inflow and outflow, only the proportion with capital control level different from the

[^4]Table 1: Comparison Between the New Capital Control Measure and the Existing Measures

|  | Tightening Inflow | Easing <br> Inflow | Tightening Outflow | Easing Outflow | Tightening Overall | Easing Overall |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of changes | 30 | 65 | 27 | 35 |  |  |
| New measure |  |  |  |  |  |  |
| Quinn (2011) |  |  |  |  | 39 | 15 |
| Observation | 333 | 333 | 333 | 333 | 333 | 333 |
| Number of changes |  |  |  |  |  |  |
| New measure | 68 | 93 | 61 | 51 |  |  |
| Fernandez et al. (2016) | 24 | 24 | 22 | 22 |  |  |
| Observation | 217 | 217 | 217 | 217 |  |  |

Note: This table reports the comparison between the newly-constructed measure of capital control changes and the existing measures. The top panel is between the new measure and Quinn, Schindler and Toyoda (2011), and the bottom panel is between the new measure and Fernández et al. (2016) index. The sample is the overlapping country-year pair between datasets.
previous year is listed. It can be drawn that the new measure captures significantly more variations than the Quinn index. For instance, the new measure records 57 country-year observations with capital control tightening ( 30 from inflow and 27 from outflow), while the Quinn index only shows 39. The difference is more drastic in capital control easing: even if only considering inflow easing, the new measure captures four times more policy changes than the Quinn index. The bottom panel lists the result on the new measure versus Fernández et al. (2016), and it shows that the stringency measure picks up at least one time more changes. In particular, on inflow tightening, which is the focus of this paper, the new measure is able to capture 68 changes while Fernández et al. (2016) displays $35 \%$ as much.

The second advantage of the new measure is that it allows to identify the maturity-dependence of capital control policy changes. This information is new to the literature, and it sheds new light on whether and how capital controls are employed in order to adjust maturity structure of external borrowing. Particularly, to examine the maturity-dependence of capital control policy is of great interest in financial crises, because it is the episode when external debt maturity exhibits potentially inefficient large movement and that policy interventions are in crucial need.

Cyclicality of Capital Inflow Controls - Employing the new measure of changes in capital controls, I now analyze the cyclicality of capital controls, that is, whether policy makers tighten or loosen capital inflow controls in financial crises relative to the pre- and post-crisis years. To do so, I provide three sets of statistics representing changes in capital inflow controls. The first one is an
indicator about whether there is any capital inflow control tightening in a specific year, the second one is the number of capital inflow control tightening policies, and the third one is the number of capital inflow control tightening policies but net the number of simultaneous capital inflow control easing.

Figure 2: Change in Capital Inflow Control Around Financial Crises

Percentage of Crises Showing Tightening



Average Number of Capital Control Tightening



Note: This figure presents the dynamics of capital inflow controls in 139 financial crisis episodes. The plot on the left of the top panel plots the share of financial crises that displaying inflow control tightening in different years of financial crisis episode. The plot on the right plots the average number of inflow tightening policies among all crisis episodes across different years of crisis. The left bottom panel is similar to the top right one but with the number of inflow tightening policy net from contemporaneous inflow easing. The right bottom panel assign weights to capital control policies, where the weights are based on the volume of the targeted flows in the previous year.

Figure 2 presents the average of the three statistics among all financial crisis episodes. The first chart in the top panel shows the proportion of crises displaying capital control tightening in each year of the financial crisis window. Taking two-year prior to the crisis as the initial state, we can observe that about $3 \%$ of the 139 financial crisis periods showing capital control tightening, and the proportion remains relative stable one year after. However, when the financial crisis hits
the economy, there is a remarkable spike in the inflow control as $12 \%$ countries impose tighter constraints on inflow, which triples the pre-crisis level. As fast as its increase, the tightening fades out rapidly and it falls down to the pre-crisis level after two years.

The second chart on the top of Figure 2 shows the number of tightening policies at each time point. Unlike the proportion measure which represents the incidence of capital control tightening in financial crisis windows, the average number of policy provides information about to what extent additional inflow constraints are employed. Starting from two-year before crisis, the average number of inflow control tightening is about 0.1 , which means on average about one capital inflow control tightening happens for every ten financial crisis episodes. In the run-up of crisis, there is still no significant tightening. However, upon the arrival of financial crisis, the average number of tightening policies jumps to one per three financial crisis episodes. After the start of the financial crisis, the tightening shifts to a declining trend and goes back to the initial level in two years.

The results of proportion and average number of capital control changes provide strong evidence for capital control tightening in financial crises, but there could be simultaneous capital control easing offsetting the tightening. To address this concern, I calculate the net average number of capital control tightening policies by subtracting the number of capital control easing out from tightening to check whether the patterns still stand. The chart in the bottom panel of Figure 2 presents the result. It is true that the contemporaneous easing policies cancel out a considerable amount of tightening policies, yet the net average number of inflow tightening continues to peak in financial crisis year. This also indicates that capital control easing in the financial crisis increases less than tightening, otherwise, it would have neutralized or even overturned the cyclicality of capital control tightening.

The net measure of capital control tightening has the merit of taking the easing counterpart into consideration, yet an implicit assumption in the measure construction is that one tightening policy can be fully offset by one easing policy. This suffers from the potential shortcoming of neglecting the heterogeneity among policies. For instance, in 2007, during the financial crisis, Slovenia passed the regulation of credit inflow easing "The required reserve ratios for deposits in foreign currency and in euros were unified at $2 \%$ for deposits with a maturity up to two years and zero for longer maturities." which indicates a credit inflow easing (for the longer maturities), while in the same year there was a capital and money market instrument regulation stating that "the issuance of capital and money market instruments by residents in the EU and nonresidents in Slovenia became subject to authorization." In the previous net capital control tightening measure, these two policies are
equally weighted (the same as the majority of existing capital control measures) which leads to a net value of zero. However, the two policies target different types of flow, and the magnitude of them may be too different to make the impact of the two policies comparable. One possible robustness check is to use the lagged value of type-specific flows as weight to each capital control policy, and its idea is to proxy the impact of the policy by the pre-policy level of the target flow. The right bottom panel of Figure 2 shows the result, and measured in weighted net number of policies, capital inflow is also tightened in crisis. Although imperfect, as the target flow reflects only one aspect of the intensity of the capital control policy, it still provides some sense of the evolution of capital control if the intensity of each policy is partially addressed.

Maturity-dependent Capital Inflow Controls - As discussed in the data construction procedure, the maturity targeting information is extracted from the narratives of capital control policy changes in two steps. The first step is to distinguish which policies contain specific maturity target (labeled as "maturity-related policy"). The second step is, for each of the maturity-related policy, differentiating whether the policy intention is to target short-term flows or long-term flows ${ }^{8}$. The short-/long-term flow information provides two important perspectives for examining the dynamics of maturity targeting of capital control policies: the proportion of maturity-related policies relative to total changes, and the proportion of short-term-targeted policies relative to total changes. The former tells whether the maturity-adjust mandate in capital controls becomes stronger or not, and the latter further pins down what is the targeted maturity if there is one.

Figure 3 provides strong evidence for the increasing maturity attention of inflow control policies in financial crises and the dominant emphasis on short-term inflow. The left-hand chart plots the proportion of maturity-related inflow tightening policy to the total number of inflow tightening policy, and it can be observed that there is a significant increase in maturity-related inflow tightening policy when financial crises take place. Specifically, when financial crisis arrives, the maturity-related policy accounts more than half of the total inflow tightening, which is more than twice of the pre-crisis level. The maturity attention is also persistent as it stays around the new level for two years after the breakout of the financial crises.

To further distinguish which maturity attracts more policy attention, the right-hand chart of Figure 3 plots the shares of the short-/long-term inflows tightening policy in total tightening policies, respectively. It can be observed that short-term flow is at the core of inflow tightening policy.

[^5]In general, short-term inflow tightening constitutes a significantly larger fraction of total inflow tightening than the long-term. In particular, in response to financial crises, the tightening policies are more and more frequently imposed to curb short-term inflows: from less than $20 \%$ prior to crisis to more than $40 \%$ upon the start of crisis and even higher after the burst of crisis. In the meantime, the share of long-term inflow tightening only shows a mild increase when crises start.

Figure 3: Maturity-dependent Capital Inflow Control


Note: This figure shows the dynamics of maturity-related inflow controls in 139 financial crisis episodes. The plot on the left plots the share of maturity-related inflow tightening policy in total tightening policy, and the plot on the right plots the share of short-/long-term inflow tightening policy in total tightening policy.

Combining the results, we can see that although inflow controls are strengthened when the financial crisis starts, the implementation of the policies is drastically different with respect to different maturities. Particularly, the controls are enhanced and pinpointed to the short-term maturity (short-term inflow targeting). This stylized pattern reveals the maturity-adjust mandate goes beyond the volume-control mandate capital inflow control policy, and it also indicates the maturity characteristics of different transactions plays an important role in the policy implementations.

Maturity-dependent Reserve Requirements - Exploring the information from the policy statements provides a meaningful way to examine the potential maturity-targeting mandate in capital inflow controls, yet essentially it is an indicator measure on whether the policy constrains short-term inflow or long-term inflow. In order to provide further insights on the extent to which the short-term capital inflow is more regulated than the long-term, I use the reserve requirements on different types of deposit from Federico, Vegh and Vuletin (2014). There are two advantages of this measure:
first, it is continuous which enables the magnitude comparison; second, for considerable amount of countries, the reserve requirement is set based on the currency and the type of deposit which helps to distinguish external borrowing with respect to different maturities. According to the categorization of deposit, I associate the reserve requirement on foreign currency term deposit as long-term external borrowing and that on foreign currency demand deposit as short-term external borrowing. ${ }^{9}$

Figure 4: Reserve Requirement on Foreign Currency Deposit


Note: This figure shows the reserve requirements on foreign currency deposit with respect to different maturities. The plot on the left shows the share of financial crises that display increase in reserve requirement, and the plot on the right presents the levels of reserve requirement on short-/long-term deposit, respectively. Here, demand deposit represents short-term inflow, and term deposit stands for long-term inflow.

Figure 4 shows the dynamics of reserve requirements in the financial crisis periods. The left panel displays the proportion of financial crises in which reserve requirement increases. Prior to crisis, there are barely any increase in reserve requirements, however, once the crisis materializes, in about $40 \%$ of crises, policy makers tighten the inflow of short-term external funds. On the contrary, less than $10 \%$ of the policy makers choose to increase long-term deposit reserve requirement. The right panel plots the levels of reserve requirement around financial crises. The pattern shows a significantly large increase in short-term deposit reserve requirement, about eight percentage points rise compared to the pre-crisis level. However, long-term deposit only experiences a minor raise in reserve requirement, with a magnitude about 4\%. Taken together, although both reserve requirements are modified upwardly during crisis, reserve requirement on short-term foreign currency

[^6]deposit is raised by a larger amount, which indicates a more substantial control is employed on the short-term inflow during financial crises.

To summarize, these facts present important patterns in financial crises about the level and the maturity composition of private external borrowing, and the capital control policy responses in terms of cyclicality and maturity targeting. The key findings include the plummet in private external borrowing, the increase in short-term borrowing, the capital inflow tightening, and the short-term inflow targeting in tightening policies. Based on these observations, the next section introduces a model of international borrowing with financial friction which is consistent with these stylized facts: private agent reduces external borrowing and shift to short-term debt during crisis, and the optimal capital control policy should tighten capital inflows constraints, particularly for short-term inflows.

## 4 The Model

This section presents a dynamic model with international borrowing in multiple maturities subject to a collateral constraint. There are three types of agent in the model: households, tradable goods producers, and international creditors. The structure of the mdoel goes as follows. The prices of short-term debt and long-term debt $q_{t}^{S}, q_{t}^{L}$ are set by international creditors and taken as exogenous by households and tradable goods producers in the small open economy. Given the cost of borrowing, tradable goods producers import tradable intermediate goods to produce, subject to a working capital constraint. Given stochastic streams of nontradable goods income $\left(y^{N}\right)$, debt prices $\left(q_{t}^{S}, q_{t}^{L}\right)$, and the rebated profit from tradable goods producers, representative households issue short- and long-term debt under collateral constraint to finance the optimal consumption allocation. Section 4.A illustrates the optimization problem of each agent, Section 4.B explains the first order conditions and defines competitive equilibrium, Section 4.C introduces Ramsey equilibrium and derives the optimal capital control policy.

## A. Environment

Household - The representative household in the economy receives utility from consumption with preference given by

$$
\begin{equation*}
\mathbb{E} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right) \tag{1}
\end{equation*}
$$

where $\beta \in(0,1)$ is the discount factor and $u(\cdot)$ is assumed to have the constant-relative-risk-aversion (CRRA) form. The total consumption $c$ is constituted by tradable goods consumption $c^{T}$ and nontradable goods consumption $c^{N}$ with Armington-type CES aggregator

$$
c_{t}=\left[\alpha\left(c_{t}^{T}\right)^{1-1 / \xi}+(1-\alpha)\left(c_{t}^{N}\right)^{1-1 / \xi}\right]^{\frac{1}{1-1 / \xi}}
$$

where $\xi$ denotes the elasticity of substitution between tradable and nontradable goods and $(1-\alpha) \in$ $(0,1)$ is the home bias.

In every period $t$, households have two sources of income. The first is an exogenous nontradable endowment $y_{t}^{N}$, and the second is the profit $\pi_{t}$ from the tradable goods production. Besides domestic income, households also have access to the global financial market by issuing short- and long-term debt. The external borrowings in both maturities are in real term, and it is denominated by tradable goods. Short-term debt is in the form of one-period discount bond. For one unit of short-term bond issued, borrowers receive $q_{t}^{S}$ unit of tradable goods and international creditors demand one unit of tradable goods repayment in the next period. Long-term bond is modeled as a perpetuity contract with deterministic infinite stream of coupons that decay geometrically at an exogenous constant rate $\delta$, following Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012). Specifically, one unit of long-term bond yields $q_{t}^{L}$ of tradable goods for the borrower upon issuance and mandates $\delta^{n-1}$ of tradable goods repayment for every future period $t+n$. Therefore, the law of motion of long-term debt repayment can be represented by

$$
d_{t+1}^{L}=\delta d_{t}^{L}+i_{t}
$$

where $d_{t}^{L}$ is the repayment for long-term debt in period $t$, and $i_{t}$ is the amount of newly-issued long-term bond in period $t$. As the bonds are non-defaultable and the borrower is in a small scale compared with the global financial market, I assume that $q_{t}^{S}$ and $q_{t}^{L}$ are exogenous to the small open economy. With these specifications, households' inter-temporal budget constraint can be written as

$$
\begin{equation*}
c_{t}^{T}+p_{t} c_{t}^{N}+d_{t}^{S}+d_{t}^{L}=\pi_{t}+p_{t} y_{t}^{N}+q_{t}^{S} d_{t+1}^{S}+q_{t}^{L}\left(d_{t+1}^{L}-\delta d_{t}^{L}\right) \tag{2}
\end{equation*}
$$

where $p_{t}$ denotes the relative price of nontradable goods in terms of tradable goods, or equivalently, the real exchange rate.

Although households can finance consumption using international financial market, the financing
is subject to frictions. I assume that the international creditors impose a collateral constraint on households as the total value of debt outstanding in each period no larger than a fraction $\kappa \in(0,1)$ of their contemporaneous income.

$$
\begin{equation*}
q_{t}^{S} d_{t+1}^{S}+q_{t}^{L} d_{t+1}^{L} \leq \kappa\left(\pi_{t}+p_{t} y_{t}^{N}\right) \tag{3}
\end{equation*}
$$

This collateral constraint is in a similar vein as Bianchi (2011) except for the inclusion of long-term debt, and it can be viewed as "maintenance margin requirement" following Mendoza and Smith (2006). Essentially, it protects the creditors from default by insuring that the face value of debt can be recovered by the market value of the pledged collateral. ${ }^{10}$

The collateral constraint affects the dynamics of the small open economy in two aspects. On one hand, it brings in financial frictions by limiting the amount external borrowing and lowering welfare compared to the frictionless case. On the other hand, it introduces pecuniary externality as households take the price of nontradable goods as exogenous when allocating consumption and borrowing, yet they neglect that the price is determined by their collective optimal consumption and borrowing choices. This externality, as will be discussed in details later, differentiates the competitive equilibrium from social planner's Ramsey equilibrium, which justifies the importance of external borrowing regulations.

Tradable Goods Producers - The tradable goods in household's consumption bundle is produced by the tradable sector using imported tradable goods $f_{t}$ using the following technology:

$$
y_{t}^{T}=\Gamma f_{t}^{\gamma}, \quad \gamma \in(0,1), \Gamma>0
$$

I assume that while producing, the producer faces a working capital constraint such that fraction $\eta$ of the input factor payment must be paid in advance before production or sale takes place, and the rest $(1-\eta)$ can be paid after the production and sale complete. In order to finance the working capital requirement, firms have to borrow $\eta f_{t}$ in units of tradable goods (the working capital) at

[^7]price $q_{t}^{S},{ }^{11}$, which will incur interest rate cost $\left(\frac{1}{q_{t}^{S}}-1\right) \eta f_{t}$. Therefore, tradable goods producers' profit maximization problem can be stated in the following way:
\[

$$
\begin{equation*}
\max _{f_{t}} \quad \pi_{t}=\Gamma f_{t}^{\gamma}-f_{t}-\left(\frac{1}{q_{t}^{S}}-1\right) \eta f_{t} \tag{4}
\end{equation*}
$$

\]

The tradable production sector with working capital constraint connects the economy's income to the international financial market conditions, by doing so it provides a micro-foundation for otherwise exogenously imposed dependence between $y$ and $q .{ }^{12}$

International Creditors - I assume risk-averse international creditors and their stochastic discount factor takes the form of Ang and Piazzesi (2003). ${ }^{13}$

$$
\begin{aligned}
\ln M_{t, t+1} & =-\phi_{0}-\phi_{1} x_{t}-\frac{1}{2} \zeta_{t}^{2} \sigma_{x}^{2}-\zeta_{t} \epsilon_{x, t+1} \\
\zeta_{t} & =\phi_{0}^{\zeta}+\phi_{1}^{\zeta} x_{t} \\
x_{t+1} & =\phi_{0}^{x}\left(1-\phi_{1}^{x}\right)+\phi_{1}^{x} x_{t}+\epsilon_{x, t+1}
\end{aligned}
$$

where $x_{t}$ is the factor in determining international investor's pricing kernel, $\zeta_{t}$ is the time-varying market price of risk associated with the sources of uncertainty $\epsilon_{x}$. Examples of the shocks to international creditor's stochastic discount factor could be time preference change, world-wide demand shifter, or uncertainty shocks. Note that, if $\lambda_{t}$ is zero for all $t$, which means the price of risk is nil, then international creditors are risk-neutral. To understand why international creditors are risk-averse even they are able to receive repayments in any state, it is key to note that the effective value of long-term debt repayment varies with states. For short-term debt, international creditors will be repaid by one unit in each case next period, therefore, there is no risk. However, the effective repayment for long-term debt is state-dependent, different from short-term debt. Specifically, the

[^8]effective repayment, which is equal to the present value of all future repayments, is $\left(1+\delta q_{t+1}^{L}\right)$, which hinges on the future state in period $t+1$. In other words, the effective repayment will be different based on different values of stochastic discount factor, and this connection between $M_{t, t+1}$ and $\left(1+\delta q_{t+1}^{L}\right)$ yields risk aversion. Essentially, international creditors' stochastic discount factor generates a time-varying term premium between short- and long-term debt, and as will be shown in the next section, time-varying term premium is critical in the model for interior optimal portfolio solution and the relative magnitude between short- and long-term capital control taxes.

## B. Competitive Equilibrium

To define the competitive equilibrium, I begin with the optimal condition for the tradable goods producers' static profit maximization problem. Producers choose the amount of imported tradable goods to maximize profit according to (4), taking $q_{t}^{S}$ as given. The producer's first order condition requires:

$$
\gamma \Gamma f_{t}^{\gamma-1}=1+\left(\frac{1}{q_{t}^{S}}-1\right) \eta
$$

It further yields the optimal level of profit

$$
\begin{equation*}
\pi_{t}=\Gamma(1-\gamma)\left[\frac{1+\left(\frac{1}{q_{t}^{S}}-1\right) \eta}{\gamma \Gamma}\right]^{\frac{\gamma}{\gamma-1}} \tag{5}
\end{equation*}
$$

The households' problem is to choose $\left\{c_{t}^{T}, c_{t}^{N}, d_{t+1}^{S}, d_{t+1}^{L}\right\}$ to maximize the present discounted value of life-time utility in (1) subject to budget constraint (2) and collateral constraint (3), given tradable goods production profit (5), exogenous nontradable endowment $y_{t}^{N}$, and bond prices $q_{t}^{S}, q_{t}^{L}$. The corresponding first order conditions are:

$$
\begin{align*}
u_{T t}^{\prime} & =\lambda_{t}  \tag{6}\\
p_{t} & =\frac{1-\alpha}{\alpha}\left(\frac{c_{t}^{T}}{c_{t}^{N}}\right)^{1 / \xi}  \tag{7}\\
\lambda_{t} q_{t}^{S}-\mu_{t} q_{t}^{S} & =\beta \mathbb{E}_{t} \lambda_{t+1}  \tag{8}\\
\lambda_{t} q_{t}^{L}-\mu_{t} q_{t}^{L} & =\beta \mathbb{E}_{t}\left[\lambda_{t+1}\left(1+\delta q_{t+1}^{L}\right)\right]  \tag{9}\\
\mu_{t}\left[\kappa\left(\pi_{t}+p_{t} y_{t}^{N}\right)-q_{t}^{S} d_{t+1}^{S}-q_{t}^{L} d_{t+1}^{L}\right] & =0, \quad \text { with } \mu_{t} \geq 0  \tag{10}\\
d_{t+1}^{S}+d_{t+1}^{L} & \leq \kappa\left(\pi_{t}+p_{t} y_{t}^{N}\right) \tag{11}
\end{align*}
$$

where $\beta^{t} \lambda_{t}$ is the Lagrangian multiplier for budget constraint and $\beta^{t} \mu_{t}$ is the Lagrangian multiplier associated with collateral constraint. Equation (6) requires the marginal utility of tradable consumption equal to the shadow value of current income. Equation (7) determines the relative price of tradable goods by marginal rate of substitution between tradable goods and nontradable goods consumption. Equation (8) and (9) are the Euler equations associated with short-term debt and long-term debt. Because of different debt prices and repayment schedule, the short-term and long-term Euler equations differ from each other in both the contemporaneous benefit of one unit of debt and the expected cost of the associated future repayment. Compared to the scenario with financial frictions, collateral constraint lays a wedge between the present value of consumption and the expected value of future repayment for both short-term debt and long-term debt. Specifically, as shown in Equation (8) and (9), when the financial crisis occurs, meaning collateral constraint binds, the shadow price of collateral constraint penalizes extra consumption through borrowing in both maturities. Finally, equation (10) and (11) state the slackness conditions of collateral constraint.

Because nontradable goods can only be consumed domestically and that there is no heterogeneity among households, market clear conditions are:

$$
\begin{align*}
c_{t}^{N} & =y_{t}^{N}  \tag{12}\\
c_{t}^{T}+d_{t}^{S}+d_{t}^{L} & =\pi_{t}+q_{t}^{S} d_{t+1}^{S}+q_{t}^{L}\left(d_{t+1}^{L}-\delta d_{t}^{L}\right) \tag{13}
\end{align*}
$$

Definition 1. (Competitive Equilibrium) The competitive equilibrium is defined as a set of processes $\left\{c_{t}^{T}, c_{t}^{N}, d_{t+1}^{S}, d_{t+1}^{L}, p_{t}, \pi_{t}, \lambda_{t}, \mu_{t}\right\}_{t=0}^{\infty}$ satisfying optimal conditions (5) to (13), given exogenous processes $\left\{y_{t}^{N}, q_{t}^{S}, q_{t}^{L}\right\}_{t=0}^{\infty}$ and initial debt levels $d_{0}^{S}, d_{0}^{L}$.

## C. Ramsey Equilibrium and Optimal Capital Controls

One important feature of the competitive equilibrium is that agents take the market prices as given, particularly in choosing debt portfolio, households take the collateral price (nontradable goods price) as given. Unlike the household in the competitive equilibrium, social planner recognizes how the aggregate variables, especially nontradable goods price, is determined in equilibrium and takes it into account when determining the optimal external financial decisions. Given this difference, it is important to compare private agents' decision and social planner's choice, and this section explains social planner's optimization problem and its connection to the competitive equilibrium.

I assume a benevolent social planner who maximizes the life-time utility of households. The
planner can choose debt portfolio for households and let the consumption portfolio and market price to be determined in the competitive way. Therefore, the Ramsey equilibrium from social planner's optimization problem can be defined as follows.

Definition 2. (Ramsey Equilibrium) The Ramsey equilibrium is defined as a set of processes $\left\{c_{t}^{T}, c_{t}^{N}, d_{t+1}^{S}, d_{t+1}^{L}, p_{t}, \pi_{t}\right\}_{t=0}^{\infty}$ maximizing the present discounted utility (1) subject to collateral constraint (3), pricing rule (7), and market clear conditions (12) and (13), given exogenous processes $\left\{y_{t}^{N}, q_{t}^{S}, q_{t}^{L}\right\}_{t=0}^{\infty}$ and initial debt levels $d_{0}^{S}, d_{0}^{L}$.

Ramsey equilibrium and competitive equilibrium share a number of optimal conditions, while they significantly differ from each other in choosing debt portfolio. To shed further light on the difference, it would be useful to analyze the Euler equations ( $8^{\prime}$ ) and ( $9^{\prime}$ ) in Ramsey equilibrium.

$$
\begin{align*}
& \left(\widetilde{u}_{T t}^{\prime}+\widetilde{\mu}_{t} \Phi_{t}\right) q_{t}^{S}-\widetilde{\mu}_{t} q_{t}^{S}=\beta \mathbb{E}_{t}\left(\widetilde{u}_{T t+1}^{\prime}+\widetilde{\mu}_{t+1} \Phi_{t+1}\right)  \tag{8’}\\
& \left(\widetilde{u}_{T t}^{\prime}+\widetilde{\mu}_{t} \Phi_{t}\right) q_{t}^{L}-\widetilde{\mu}_{t} q_{t}^{L}=\beta \mathbb{E}_{t}\left[\left(\widetilde{u}_{T t+1}^{\prime}+\widetilde{\mu}_{t+1} \Phi_{t+1}\right)\left(1+\delta q_{t+1}^{L}\right)\right] \tag{9'}
\end{align*}
$$

where $\widetilde{u}$ is households' marginal utility, $\widetilde{\mu}$ is the shadow price of collateral, and $\Phi_{t} \equiv \kappa y_{t}^{N} \frac{\partial \widetilde{p}_{t}}{\partial \widetilde{c}_{t}^{T}}$, representing the change in collateral value in response to tradable consumption.

Compared with (8), Euler equation ( $8^{\prime}$ ) contains one additional term $\mathbb{E}\left(\widetilde{\mu}_{t+1} \Phi_{t+1}\right)$ in the evaluation of expected future repayment cost. ${ }^{14}$ Why is this term neglected by the private agents? Fundamentally, it is because of the pecuniary externality stemmed from the presence of the key market price $\left(p^{N}\right)$ in collateral constraint, which is taken as exogenous by private agents. When the collateral constraint binds in the future, i.e. $\widetilde{\mu}_{t+1}>0$, debt repayment is going to trigger a contractionary spiral through the feedback loop of consumption decrease, collateral price fall, and borrowing capacity shrink. However, private agents treat collateral price as given, therefore they fail to internalize that the feedback loop generates financial amplification effect of debt repayment, resulting in undervaluing the expected cost of future debt repayment. The magnitude of pecuniary externality is determined by the probability of future collateral constraint binding ( $\widetilde{\mu}_{t+1}$ ) and the financial amplification effect when collateral constraint binds $\left(\Phi_{t+1}\right)$.

Similar to short-term debt's Euler equation, the long-term debt's counterpart (9') also includes an extra term in the expect future cost of debt issuing, $\mathbb{E}_{t} \widetilde{\mu}_{t+1} \Phi_{t+1}\left(1+\delta q_{t+1}^{L}\right)$. However, different

[^9]from short-term debt which requires one unit of repayment, one unit of long-term debt demands $\left(1+\delta q_{t+1}^{L}\right)$ units of "effective" repayment at $t+1 .{ }^{15}$ Therefore, the associated pecuniary externality is $\left(1+\delta q_{t+1}^{L}\right)$ times as of short-term debt in each future collateral constrained state.

Given the difference between competitive equilibrium and Ramsey equilibrium, a natural question would be whether the Ramsey equilibrium allocation can be supported as a regulated competitive equilibrium or not. To analyze the optimal policy intervention, I follow the literature (i.e. Korinek (2017), Bianchi (2011), etc.) and assume that social planner has two policy tools: tax on short-term borrowing and tax on long-term borrowing, and that the total tax revenue is rebated to household in a lump-sum way. Specifically, households maximize the present discounted value of life-time utility given capital control taxes, and the optimization problem can be formalized as follows.

Definition 3. (Competitive Equilibrium Under Regulation) The competitive equilibrium under capital control policies is defined as a set of processes $\left\{c_{t}^{T}, c_{t}^{N}, d_{t+1}^{S}, d_{t+1}^{L}, p_{t}, \pi_{t}, \lambda_{t}, \mu_{t}\right\}_{t=0}^{\infty}$ satisfying optimal conditions (5) to (13), given exogenous processes $\left\{y_{t}^{N}, q_{t}^{S}, q_{t}^{L}\right\}_{t=0}^{\infty}$, initial debt levels $d_{0}^{S}, d_{0}^{L}$, and capital control taxes $\left\{\tau_{t}^{S}, \tau_{t}^{L}\right\}_{t=0}^{\infty}$ maximizing the present discounted utility (1) subject to collateral constraint (3) and budget constraint

$$
\begin{equation*}
c_{t}^{T}+p_{t} c_{t}^{N}+d_{t}^{S}+d_{t}^{L}=\pi_{t}+p_{t} y_{t}^{N}+\left(1-\tau_{t}^{S}\right) q_{t}^{S} d_{t+1}^{S}+\left(1-\tau_{t}^{L}\right) q_{t}^{L}\left(d_{t+1}^{L}-\delta d_{t}^{L}\right)+T_{t} \tag{2'}
\end{equation*}
$$

where $T_{t}=\tau_{t}^{S} q_{t}^{S} d_{t+1}^{S}+\tau_{t}^{L} q_{t}^{L}\left(d_{t+1}^{L}-\delta d_{t}^{L}\right)$.
In order to decentralizing the Ramsey equilibrium, social planner chooses taxes to maximize household's present discounted utility while allowing debt portfolio to be chosen by the private agents given the after-tax debt price, and that goods market clear competitively under collateral constraint. Accordingly, the optimization problem can be defined as follows.

Definition 4. (Ramsey Optimal Competitive Equilibrium) The Ramsey optimal competitive equilibrium is defined as a set of processes $\left\{\tau_{t}^{S}, \tau_{t}^{L}, c_{t}^{T}, c_{t}^{N}, d_{t+1}^{S}, d_{t+1}^{L}, p_{t}, \pi_{t}, \lambda_{t}, \mu_{t}\right\}_{t=0}^{\infty}$ maximizing the present discounted utility (1) subject to pricing rule (7), slackness conditions (10) and (11), market clear conditions (12) and (13), and Euler equations under regulation ( 8 ") and ( 9 "), given exogenous

[^10]processes $\left\{y_{t}^{N}, q_{t}^{S}, q_{t}^{L}\right\}_{t=0}^{\infty}$ and initial debt levels $d_{0}^{S}, d_{0}^{L}$.
\[

$$
\begin{align*}
& \lambda_{t} q_{t}^{S}\left(1-\tau_{t}^{S}\right)-\mu_{t} q_{t}^{S}=\beta \mathbb{E}_{t} \lambda_{t+1} \\
& \lambda_{t} q_{t}^{L}\left(1-\tau_{t}^{L}\right)-\mu_{t} q_{t}^{L}=\beta \mathbb{E}_{t}\left[\lambda_{t+1}\left(1+\delta\left(1-\tau_{t+1}^{L}\right) q_{t+1}^{L}\right)\right] \tag{9"}
\end{align*}
$$
\]

Proposition 1. (Optimal Capital Controls) The Ramsey optimal allocation can be decentralized in a competitive equilibrium with taxes on short- and long-term debt satisfying

$$
\begin{align*}
\tau_{t}^{S} & =1-\beta \cdot \frac{\mathbb{E} \tilde{\lambda}_{t+1}}{q_{t}^{S} \widetilde{\lambda}_{t}}  \tag{14}\\
\widetilde{\lambda}_{t} q_{t}^{L}\left(1-\tau_{t}^{L}\right) & =\beta \mathbb{E}_{t}\left[\widetilde{\lambda}_{t+1}\left(1+\delta\left(1-\tau_{t+1}^{L}\right) q_{t+1}^{L}\right)\right] \tag{15}
\end{align*}
$$

where $\tilde{\lambda}$ is the optimal shadow price of income in Ramsey equilibrium.
The optimal capital control policy (see proof in Appendix A) follows the same logic as SchmittGrohé and Uribe (2017) that social planner can always choose tax rates such that individual agents' trade-off between contemporaneous benefit of borrowing and the expected future cost of repayment coincides with that of social planner. Essentially, since the future cost of repayment is undervalued by private agents, capital control taxes on external borrowing are set to decrease the benefit of borrowing to balance out the miscalculation of cost.

Summary of the Model - The model differs from a standard collateral constraint model in small open economy from two aspects: first, instead of one short-term debt, the model grants borrower the access to long-term debt, which introduces maturity choice in equilibrium; second, international creditor is risk-averse, which generates term premium in external borrowing. The economy is driven by stocks in international creditor's stochastic discount factor, whose dynamics will affect the borrowing cost faced by the small open economy, and consequently, total debt position, maturity structure, and consumption portfolio. The inefficiency in private agents' debt maturity structure due to pecuniary externality gives room for policy intervention, and the state-contingent and maturity-dependent optimal policy is able to correct pecuniary externality and restore social optimum.

## 5 Quantitative Analysis

This section presents the quantitative analysis of the model. Section 5.A describes the parameterization and compares the key statistics from the competitive economy simulation to the data. Section 5.B discusses the trade-offs that determines the debt portfolio choice. Section 5.C analyzes the cyclicality of optimal capital control polices. Section 5.D inspects the difference between short- and long-term capital controls. Section 5.E studies the optimal debt portfolio in competitive equilibrium and Ramsey equilibrium. Section 5.F evaluates the welfare effect of optimal capital controls.

## A. Parameterization and Key Statistics

I calibrate the model to Argentine data (1983 - 2001) with the time unit as one year. Table 2 shows the values of parameters. ${ }^{16}$ Following Bianchi (2011), the risk aversion level is set at 2, the elasticity of substitution between tradable goods and nontradable goods is set at 0.83 , the weight on tradables in CES aggregator is set at $31 \%$, and the collateral constraint tightness $\kappa$ is set at 0.32. The time discount factor $\beta$ is set to be 0.86 to keep $\beta / \bar{q}^{S}$ at the same value as Bianchi (2011). The decaying rate of long-term bond is set at 0.90 to match the average duration of private sector external borrowing in the data.

The international creditors' stochastic discount factor (SDF) and tradable sector production are new to the literature, and they are calibrated in the following way. For the SDF, first, I choose the factor $x_{t}$ as the Moody's Seasoned Baa Corporate Bond Spread as the proxy for global risk, and estimate the $\operatorname{AR}(1)$ process to obtain $\phi_{0}^{x}, \phi_{1}^{x}$ and $\sigma_{x}^{2}$. Global risk has been shown as an important factor in driving the variation in interest rate and key macro variables in emerging economy. For instance, Akıncı (2013) finds that global financial risk shocks explain about $20 \%$ of movements both in the country spread and in the aggregate activity in emerging economies. To shed further light on the relationship between global risk and the Argentine interest rates, Figure 5 plots the quarterly time series of Baa spread. In particular, in the three financial crises (1989, 1004, 2001, denoted by the vertical dashed line), Baa spread significantly rose. Moreover, Moody's Baa spread has considerable large explanation power for the variation in Argentine interest rate: a simple regression $\ln \left(1+i_{t}\right)=\alpha+\beta \ln \left(1+\right.$ spread $\left._{t}\right)+\epsilon_{t}$ yields $\widehat{\beta}$ significant at level $0.1 \%$ and $R^{2}=20.1 \%$.

Second, based on the affine property $-q_{t}^{S}=\phi_{0}+\phi_{1} x_{t}$, I use the short-term interest rate from Neumeyer and Perri (2005) to estimate $\phi_{0}$ and $\phi_{1}$. Lastly, $\phi_{0}^{\zeta}$ and $\phi_{1}^{\zeta}$ are set to match the mean of

[^11]Figure 5: Term Premium: Calibration vs. Data


Note: This figure plots quarterly sequence of Moody's Baa corporate spread (right axis) and Argentine short-term interest rate (left axis) from Neumeyer and Perri (2005) from 1987Q1 to 2001Q1.
debt-to-GDP ratio and the mean of short-term debt share. For the tradable goods production block, $\Gamma, \gamma, \eta$ are set to match the median, standard deviation of tradable sector output and its correlation with short-term interest rate.

Table 2: Parameter Values

| Parameter | Description | Value |
| :--- | :--- | :--- |
| $\sigma$ | Risk aversion | 2 |
| $\xi$ | Elasticity of substitution | 0.83 |
| $\alpha$ | Weight on tradables in CES | 0.31 |
| $\beta$ | Discount factor | 0.86 |
| $\kappa$ | Collateral constraint | 0.32 |
| $\delta$ | Coupon decaying rate | 0.90 |
| $\Gamma$ | Tradable goods production function | 2.11 |
| $\gamma$ | Tradable goods production function | 0.83 |
| $\eta$ | Working capital constraint in tradable production | 0.34 |
| $\phi_{0}^{x}, \phi_{1}^{x}, \sigma_{x}^{2}$ | AR(1) coefficients of the factor in pricing kernel | $[0.02,0.89,0.035]$ |
| $\phi_{0}^{\zeta}, \phi_{1}^{\zeta}, \phi_{0}, \phi_{1}$ | international creditor SDF | $[0.68,0.31,0.97,0.96]$ |

Since risk-averse international creditor is one of the key ingredients that differ the model from
the literature, it would be useful to compare the term structure derived from the calibration to the data to see whether the model is well grounded by the reality or not. Figure 6 plots the relationship between 12 -year over 3 -year term premium and 3 -year spread ${ }^{17}$ for both data and calibration. It shows that the two both generate a positive correlation, which indicates that term premium rises when borrowing rate is high, i.e. bad states. Moreover, the slopes are considerable close. Since the model is calibrated to private sector external borrowing and the data is from sovereign bond term structure, one possible explanation for the difference between the slopes could be the corporate sector spread over sovereign.

Figure 6: Term Premium: Calibration vs. Data
Term Premium: 12-year vs. 3-year Sovereign Bonds


Note: This plot shows the relationship between term premium in sovereign bond and Moody's Baa corporate spread. The raw date is in blue, and the model indicated relationship is the dashed black line. Sovereign bond term structure data is from Broner, Lorenzoni and Schmukler (2013).

The competitive equilibrium is solved using Euler equation iteration, and the Ramsey equilibrium applies value function iteration. To evaluate the performance of the model, Table 3 reports the key second moments from data and in the simulated competitive equilibrium. The model is able to generate qualitatively similar moments to the data.

[^12]Table 3: Statistics: Model and Data

| Nontargeted | Model | Data | Targeted | Model | Data |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Stdev total consumption | 4.8 | 6.2 | debt/GDP | 29.3 | 30.6 |
| Stdev real exchange rate | 4.8 | 8.2 | short-term debt/debt | 23.2 | 20.3 |
| Stdev trade balance to GDP | 2.7 | 2.4 | $\operatorname{median} y^{T}$ | 0.02 | 0.01 |
| $\operatorname{Corr}(G D P, c)$ | 0.92 | 0.88 | $\operatorname{Stdev}\left(y^{T}\right)$ | 0.13 | 0.11 |
| $\operatorname{Corr}(G D P$, real exchange rate $)$ | 0.81 | 0.41 | $\operatorname{Corr}\left(y^{T}, \ln \frac{1+r^{S}}{1+\bar{r}^{S}}\right)$ | -0.81 | -0.87 |
| $\operatorname{Corr}(G D P$, trade balance $/ G D P)$ | -0.74 | -0.84 |  |  |  |

## B. Debt Portfolio

This section analyzes the key forces and the trade-offs faced by the borrowers that determine the optimal debt portfolio. Essentially, the optimal portfolio is shaped by two sets of trade-offs: borrowing benefit versus repayment cost, and short-term versus long-term borrowing. The first trade-off is inter-temporal which is the standard dynamic consumption allocation. The second trade-off is intra-temporal that is based on the cost benefit of short-term borrowing and the insurance benefit of long-term borrowing.

Borrowing Benefit versus Repayment Cost - When determining the total borrowing, the borrower makes sure that the marginal benefit of borrowing and consuming today is equal to the expected marginal future cost of repayment. Specifically, it can be represented by the Euler equation of consumption.

$$
u_{T t}^{\prime} q_{t}^{S}-\mu_{t} q_{t}^{S}=\beta \mathbb{E}_{t} u_{T t+1}^{\prime}
$$

where $\mu_{t}$ is the Lagrangian multiplier of collateral constraint. Different from the frictionless framework, the presence of collateral constraint punishes extra borrowing if total borrowing already reaches the limit, and this wedge decreases the marginal utility of borrowing ceteris paribus. As a result, private agents borrow less than if there is no friction in borrowing. In other words, private agents realize that the desired future borrowing might be unavailable due to collateral constraint, therefore they accumulate precautionary saving to cope with the constrained borrowing capacity in future bad states. Taken together, collateral constraint generates underborrowing than the frictionless counterpart. Figure 7 confirms that the ergodic distribution of total borrowing in the competitive equilibrium with collateral constraint assigns a higher probability to lower levels of debt.

Figure 7: Ergodic Distribution of Total Debt: Friction versus Frictionless
Total Debt


Note: This plot presents the ergodic distribution of total debt in the frictionless economy and in the economy with collateral constraint, respectively.

Insurance Benefit versus Cost Benefit - When determining the maturity structure of debt, private agents make sure that, given the total amount of debt, the subjective discounted expected future repayment cost is the same no matter borrowing short-term debt or long-term debt. To analyze the benefit and cost of deviating from the optimal maturity structure by swapping shortand long-term debt, I compare all the portfolios that yields the optimal consumption level (i.e. keeping total debt position unchanged) in the current period. ${ }^{18}$ As the debt price is exogenous, these portfolios maintain a constant ratio between the quantities of debt equal to the relative price of debt. If presented in the $\left(d_{t+1}^{S}, d_{t+1}^{L}\right)$ quadrant, essentially, these portfolios constitute a straight line.

$$
q_{t}^{S} d_{t+1}^{S}+d_{t}^{L} d_{t+1}^{L}=y_{t}^{T}-c_{t}^{T}-d_{t}^{S}-\left(1+\delta q_{t+1}^{L}\right) d_{t}^{L}
$$

To evaluate these portfolios, since they yield the same debt and consumption in the current period, it would be sufficient to compare their present discounted cost of future repayments. To facilitate

[^13]the comparison, I restate the borrower's optimization problem in the following recursive way.
\[

$$
\begin{array}{rl}
V\left(d_{t}^{S}, d_{t}^{L}, \mathbf{S}_{t}\right)=\max _{c_{t}^{T}, d_{t+1}^{S}, d_{t+1}^{L}} & u\left(c_{t}\right)+\beta \mathbb{E}_{t} V\left(d_{t+1}^{S}, d_{t+1}^{L}, \mathbf{S}_{t+1}\right) \\
& s . t . \\
& c_{t}^{T}+d_{t}^{S}+d_{t}^{L}=\pi_{t}+q_{t}^{S} d_{t+1}^{S}+q_{t}^{L}\left(d_{t+1}^{L}-\delta d_{t}^{L}\right) \\
& q_{t}^{S} d_{t+1}^{S}+q_{t}^{L} d_{t+1}^{L} \leq \kappa\left(\pi_{t}+p_{t} y_{t}^{N}\right) \\
& \pi_{t}=\Gamma(1-\gamma)\left[\frac{1+\left(\frac{1}{q_{t}^{S}}-1\right) \eta}{\gamma \Gamma}\right]^{\frac{\gamma}{\gamma-1}}
\end{array}
$$
\]

where $\mathbf{S}_{t}$ represents the current exogenous states $\left\{q_{t}^{S}, q_{t}^{L}, y_{t}^{N}\right\}$.
Consider the change in the present discounted value of future utility for a portfolio that deviates from the optimal maturity structure with replacing short-term debt with long-term debt. The overall change stems from two sources: the increase in $d_{t+1}^{L}$ and the decrease in $d_{t+1}^{S}$. Applying envelope theorem, the change in households' life-time utility takes the form as following

$$
\left.\frac{d \mathbb{E}_{t} V\left(d_{t+1}^{S}, d_{t+1}^{L}, \mathbf{S}_{t+1}\right)}{d d_{t+1}^{L}}\right|_{c_{t}^{T}}=\mathbb{E}_{t}\left(u_{T t+1}^{\prime} \cdot \frac{q_{t}^{L}}{q_{t}^{S}}\right)-\mathbb{E}_{t}\left[u_{T t+1}^{\prime}\left(1+\delta q_{t+1}^{L}\right)\right]
$$

The first term on the right-hand side represents the utility gain associated with the decrease in short-term debt repayment and the second term is the utility loss due to the increase in long-term debt repayment. The key difference between the two terms is that the increase in long-term debt repayment $\left(1+\delta q_{t+1}^{L}\right)$ depends on future states while the decrease in short-term debt repayment is pre-determined in period $t$, and this difference leads to the insurance benefit of the long-term and the cost benefit of the short-term, respectively.

To gain more insights in the trade-off between insurance and cost, I analyze these two terms in turns. First, long-term debt provides insurance benefit, and the insurance is grounded by the negative correlation between borrower's marginal utility and the repayment cost of long-term debt. Specifically, in future bad states, borrowing rate increases, which aggravates the situation as private agents face a high borrowing cost (low $q$ ), and consumption is low, leading to high marginal utility as private agents are in great needs of consumption. In the meantime, However, the long-term debt repayment, which effectively is equal to $\left(1+\delta q_{t+1}^{L}\right)$, decreases in future bad states. ${ }^{19}$ Therefore, although the high borrowing cost makes it difficult for the borrower to obtain external financial resource, it reduces the repayment of long-term debt at the same time. In other words, long-term

[^14]debt helps to alleviate the adverse future states by mandating low repayment, which provides insurance for the borrowers.

Second, as the counterforce against long-term debt's insurance benefit, the low average borrowing cost of short-term debt favors short-term liability accumulation. Keeping everything else unchanged, consider an extreme case that a negative shock in the current period and the spread between shortand long-term debt spikes, as a result, long-term debt price converges to the short-term. In this scenario, it would be overwhelmingly costly to issue long-term debt as it provides the same funds as the short-term in the current period while demands more repayment in the future.

Figure 8: Insurance Benefit and Borrowing Cost of Long-term Debt Accumulation


Note: This chart plots the trade-off associated with swapping short-term debt with long-term keeping current consumption unchanged. The vertical axis is the net benefit of the swap, and the horizontal axis is the Moody's Baa spread. The larger the spread is, the worse borrowing condition faced by the borrowers. The initial debt is set as the median in simulation.

Figure 8 shows the trade-off quantitatively, and it presents the net benefit of switching to long-term debt on the margin. In the bad states of the next period (high spread, difficult to borrow from the rest of the world), the repayment is low, which effectively transfers resources to the future bad state and increases utility. In the good states of the next period, the low spread induces low borrowing cost, and the resulted high repayment of long-term debt induces utility loss. With the optimal debt portfolio, considering both the good and bad states in the next period, the insurance benefit and the cost benefit should be equal in expected term, which means that the weighted average of the values along the solid line is zero.

Having inspected the debt portfolio choice, one natural question is: what is the role of collateral
constraint in shaping the maturity structure? It can be observed that with or without collateral constraint, the optimal intra-temporal condition remains the same. Therefore, collateral constraint does not directly influence the insurance versus cost trade-off. However, collateral constraint still affects maturity structure, and its impact is indirectly through affecting the total debt position. The reason is that keeping current utility constant requires total debt staying the same, which means no matter how the composition of debt changes it will not invoke changes in collateral constraint. As a result, this intra-temporal trade-off works in the same way as in the portfolio choice without financial friction. However, as illustrated before, collateral constraint plays a role in the inter-temporal trade-off and generates precautionary saving and under-borrowing compared with frictionless case. Given that maturity composition of debt depends on how much debt is taken in total, hence, maturity changes along with overall debt position. Taken together, collateral constraint directly alters aggregate amount of debt, as a side effect, maturity structure also changes.

Figure 9: Ergodic Distribution of Maturity: Friction versus Frictionless
Share of Short-term Debt


Note: This plot shows the ergodic distribution of short-term debt share in the frictionless economy and the economy with collateral constraint, respectively.

Figure 9 shows how collateral constraint affects the optimal portfolio choice compared with the frictionless scenario. In the absence of collateral constraint, the maturity is significantly shorter compared to that with friction. This difference is resulted from both the low cost of short-term debt and the insurance benefit of long-term debt. On one hand, as short-term debt is relatively cheaper to borrow as indicated by the data (high $q^{S}$, or low $r^{S}$, equivalently), borrower prefers accumulating short-term debt. However, issuing short-term debt quickly consumes collateral due to
high $q^{S}$, therefore, in a friction world, borrower has to sacrifice the low cost of short-term debt to fulfill the collateral requirement. On the other, as illustrated before, collateral constraint leads to precautionary saving, which is essentially a buffer for future adverse shocks. The insurance benefit of long-term debt aligns well with the precautionary motive as it helps to transfer resources to future bad states, therefore private agents borrow more in long-term debt so as to cope with adverse shocks tomorrow.

## C. Optimal Capital Control Policy in Financial Crisis

This section considers how social planner sets optimal capital control in response to financial crises. As discussed in Section 4.C, the presence of collateral constraint, in particular the dependence of collateral value on nontradable goods price, induces a wedge between the expected future cost of repayment perceived by social planner and by individual agents. To correct the wedge, according to Proposition 1, social planner can employ taxes on external borrowing to decentralize the Ramsey equilibrium as a regulated competitive equilibrium. The rest of the section will provide further insights about the cyclicality of short- and long-term capital controls.

To analyze the optimal capital control in coping with financial crisis, I start with defining the crisis period in the model. Following Schmitt-Grohé and Uribe (2017), I define the financial crisis period in the model as the periods when the collateral constraint binds. To characterize the dynamics in financial crisis, I simulate the competitive economy for one million years. The procedure yields 84,525 crisis periods (per 11.8 years), and I select a seven-year window (three-year lead and three-year lag) for each crisis. Using the same stochastic processes, I simulate the Ramsey problem for one million years, and Ramsey equilibrium generates 37,980 collateral constraint binding incidents (per 26.3 years), which confirms that the pecuniary externality leads to more financial crises.

Figure 10 presents the evolution (average across all crises windows) of the key aggregate variables around financial crisis. In general, financial crises in competitive equilibrium are triggered by plummet in debt prices, or equivalently, spike in external interest rates. The hike in borrowing cost depresses tradable goods production and leads to a collapse in total income. Following high borrowing cost and low income, tradable consumption decreases, and real exchange rate depreciates. On the financial side, the increase in borrowing cost makes it difficult for the economy to issue debt. Following the shrink in the tradable goods production and the drop of nontradable goods price, the value of collateral falls and further reduces the borrowing. Although both short- and long-term debt

Figure 10: Dynamics in Financial Crises


Note: This chart plots the evolution of key aggregate variables in financial crisis window. The horizontal axis is the number of years away from crisis. Solid line represents competitive economy, and dashed line is Ramsey economy.
price slump, the short-term stays relatively cheaper to borrow on average. As a result, the external borrowing tilts to short-term during crisis. Since there is less external financial resource that can be employed to consume, trade balance improves.

The dynamics in the Ramsey economy shares a similar trend with the competitive counterpart, however, it shows significantly less severity. To begin with, in the Ramsey economy, the borrower enters the crisis zone with a substantially lower debt level than that in the competitive equilibrium. As a result, the borrower has higher income net debt repayment at disposal and hence suffers less from the adverse shocks in external borrowing. Therefore, the economy experiences a relatively smaller scale contraction in output and consumption, a more moderate depreciation in real exchange rate, and maintains a slightly higher collateral value, external borrowing, and continues the trade deficit. Overall, the financial crises bring less disturbance to the Ramsey economy.

The most drastic difference between the Ramsey equilibrium and the competitive equilibrium comes from the financial side, especially on total borrowing, collateral value, and maturity structure. In crisis, collateral value in Ramsey equilibrium is much higher than that of competitive equilibrium, and more sufficient collateral value grants larger borrowing capacity which plays a significant role in easing the severity of crises. Furthermore, the Ramsey economy selects an optimal portfolio with larger proportion of long-term debt than the competitive equilibrium counterpart and maintains a higher overall maturity upon the occurrence of crisis. The larger weights on long-term debt benefits the Ramsey economy in two aspects. First, as discussed before, the long-debt provides insurance for the economy against negative shocks, hence it helps to cope with the crisis. Second, with higher share of long-term debt to start with, there is less flight to short-term borrowing in crisis and the relatively stable share of long-term debt continues helping the economy to survive crisis.

Table 4: Crisis Severity Comparison Between Competitive Equilibrium and Ramsey Equilibrium

|  | $\Delta \mathrm{GDP}$ | $\Delta c^{T}$ | depreciation | $\Delta$ debt | $\Delta$ tradable balance |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Competitive | $-33.8 \%$ | $-34.1 \%$ | $44.0 \%$ | $-31.8 \%$ | 0.33 |
| Ramsey | $-17.6 \%$ | $-17.7 \%$ | $20.1 \%$ | $-9.5 \%$ | 0.20 |

Note: This table reports the crisis severity comparison between competitive equilibrium and Ramsey equilibrium in terms of GDP decrease, tradable goods consumption decrease, depreciation of real exchange rate, debt decrease, and tradable balance increase.

Having established the less severity of financial crisis in Ramsey economy than that in competitive equilibrium, I now analyze the characteristics of the optimal capital control policies that decentralize Ramsey equilibrium as regulated competitive equilibrium. According to Proposition 1, the optimal capital control taxes can be derived in close form for both short- and long-term borrowing. Figure

11 presents the optimal capital control policies around financial crisis. The most prominent feature is that both capital controls are significantly tighten in crisis. Specifically, the magnitude of tax almost doubles the pre-crisis level for both short- and long-term. Furthermore, short-term inflow control shows a prominent tightening: the increase in tax rate is more than $2 \%$, while the long-term counterpart is $1.5 \%$. Despite of the extensive tightening in capital controls in crisis, the increase in capital controls is short-lived. The taxes immediately drop in one year after the crisis, they continue the decline trend and return back to the pre-crisis level in three years.

Within crisis periods, the sudden-stop periods are of great interest in analyzing external borrowing and optimal regulations. Typically, sudden-stop starts with financial crashes and severe deterioration in external financial condidtion, and it is followed by deep recessions that differ markedly from typical business cycles. Examples include Latin America Crisis (1994), East Asian Crisis (1997), and Russian Crisis (1998). In order to examine these episodes, I define sudden-stop following SchmittGrohé and Uribe (2017), I define a boom-bust episode as a window where tradable production profit starts above trend and is below trend three years later. Figure A. 3 displays the dynamics of key aggregate variables and optimal capital controls. The trend in aggregate variables is similar to crisis but changes are more pronounced. For instance, total income drops by $50 \%$ compared to $25 \%$ in standard crisis, and total debt shrinks to half of the boom size while it only decreases by $20 \%$ in standard crisis. On the policy side, optimal capital control taxes also increase in sudden stop periods, and the magnitude of capital control tightening also substantially exceeds those in standard crisis.

Figure 11: Optimal Capital Control in Financial Crisis Window


Note: This plot shows the dynamics of capital control taxes of short-/long-term debt in financial crisis episode, respectively.

To shed further light on the cyclicality of short- and long-term capital controls, it is useful to go beyond the crisis episodes. To this end, Figure 12 provides the unconditional relationship between overall income of the economy and the level of capital controls of all the states. As can be observed, there is remarkable variation in capital control taxes across different states of the economy, indicating social planner needs to actively tune capital control taxes to regulate the individual agents in order to achieve the Ramsey optimal allocation. Considering how capital controls change with respect to aggregate conditions, there are three important patterns. Firstly, capital control tax is positive even in normal times, which indicates that the optimal capital flow regulation is macroprudential. Secondly, capital controls are tighter in bad states than good states. In normal times, the capital control taxes are low in magnitude with small variation. To the contrary, the capital control taxes are higher on average and also more volatile in crisis. Thirdly, in normal time, the capital regulation policy does not show strong correlation with aggregate state. However, there is a pronounced negative relationship between the level of capital control taxes and the aggregate economy condition.

Figure 12: Cyclicality of Optimal Capital Control


Note: This plot shows the unconditional relationship between capital control tax and GDP. Short- and long-term debt taxes are plotted on the left panel and the right panel, respectively. In each plot, crisis periods are shown in red plus, and normal periods are shown in blue dot.

The previous analysis focused on the shock that introduced changes in domestic interest rates. Although the interest rate shock plays an important role in the business cycle dynamics of emerging economies, ${ }^{20}$ another equally crucial shock is the output shock. Figure A. 4 shows that with additional

[^15]shock from nontradable endowment, in financial crises, the key aggregate variables exhibit similar dynamics as that under interest rate shock. More importantly, the tax rate on short-term inflow is also larger than that on the long-term in general, and the difference is much remarkable during crises.

Inspecting the Mechanism - Essentially, capital control taxes are imposed to correct the pecuniary externality rising from the fact that private agents neglecting the contribution of their consumption/borrowing decision through the feedback loop of consumption, collateral price, and borrowing capacity. As discussed in Section 4.C, pecuniary externality results in undervaluing the expected future repayment cost, and the undervaluation is equal to $\mathbb{E}\left[\widetilde{\mu}_{t+1} \Phi_{t+1}\right]$ in the inter-temporal trade-off. Therefore, the cyclicality of short- and long-term capital control taxes is determined by the cyclicality of the magnitude of undervaluation.

The magnitude of undervaluation can be decomposed to two parts: the probability of future collateral constraint binding $\left(\widetilde{\mu}_{t+1}\right)$ and the financial amplification effect $\left(\Phi_{t+1}\right)$. The probability of binding collateral constraint affects undervaluation because it is the prerequisite of activating financial amplification. Intuitively, if the collateral constraint is slack, suggesting that the original collateral value is sufficient to support the unconstrained optimal borrowing, the marginal change in debt repayment will not affect the optimal consumption decision. Conditional on binding collateral constraint, the financial amplification effect represents the overall decrease in future utility induced by extra unit of debt repayment through the feedback loop of consumption decrease, collateral price fall, and borrowing limit contraction. Taken together, the product of these two terms stands for the undervaluation in future debt repayment cost.

When do future collateral constraint binding probability and financial amplification effect take a large value? The answer is crisis time. During normal time, due to persistent shock, the probability of binding collateral constraint in the next period is relatively low. Also, when collateral constraint is slack, typically consumption is relatively high, and price is not very responsive to demand increase which means low financial amplification effect. To the contrary, during financial crisis, the probability of continuing crisis is high (collateral constraint binds). In the meantime, with low level of consumption, small demand increase is able to generate large increase in price, which yields a significant financial amplification effect. Therefore, the undervaluation is more severe if current period is in crisis. In other words, private agents over-borrow because of undervaluing future repayment cost, especially in crisis time compared with the optimal level in Ramsey equilibrium. Therefore, it is optimal to raise capital control in crisis to lead the private agents effectively internalize
the pecuniary externality and restrain themselves from excess borrowing.

## D. Comparison Between Capital Control Policy for Short- and Long-term Borrowing

Last section examines the general pattern of capital controls and shows that the optimal capital controls are tightened in crisis which aligns well with what is observed in reality. Having established the cyclicality, this section analyzes the short- and long-term optimal capital controls individually and examines the mechanism that leads to their different patterns.

As indicated in Figure 11 and 12, short- and long-term capital controls vary from each other in both magnitude and the response to economy-wise fluctuations. To provide a closer look at the comparison, Figure 13 presents the cumulative density function of the ratio between short- and long-term capital control taxes on overall levels, $\tau^{S} / \tau^{L}$, and on the changes in crises, $\Delta \tau^{S} / \Delta \tau^{L}$, (relative to one-period prior to crisis). On the magnitude side (left panel), the tax on short-term debt is greater than the long-term counterpart almost all the time which suggests that the optimal capital controls always impose more restrictions on short-term borrowing. On the change side (right panel), the increase in short-term capital control tax constantly surpasses that in long-term which means that short-term borrowing is tightened by a larger extent in crises. Altogether, the optimal short-term capital control tax exceeds the long-term counterpart in both level and the response to crises.

Figure 13: Comparison Between Short- and Long-term Capital Controls



Note: This plot shows the cumulative density function of the ratio between short-term debt tax and long-term debt tax (on the left) and that of the ratio between the increases in short-term flow tax and long-term flow tax in crisis (on the right).

The Role of Term Premium - This part demonstrates that the positive term premium is the key that drives the distinct patterns of short- and long-term optimal capital controls. To do so, I start with the trade-off between cost benefit and insurance benefit in maturity selection to see the role of pecuniary externality. I then show that the magnitude of pecuniary externality depends largely on term premium and provide intuition that small term premium may fail to derive short-term inflow targeting. Finally, I consider a counterfactual case with small term premium and show that short-term inflow control is virtually the same as the long-term.

In the model, term premium, defined as the difference between the effective interest rate on a long-term bond and the short-term interest rate over the same period, can be expressed in the following way

$$
t p_{t}=\frac{\mathbb{E}_{t}\left(1+\delta q_{t+1}^{L}\right)}{q_{t}^{L}}-\frac{1}{q_{t}^{S}}
$$

It plays a crucial role in affecting the pecuniary externality rooted in maturity choice. To see the pecuniary externality borne in maturity choice, it would be useful to compare social planner's optimal condition for maturity choice trade-off with private agents' counterpart. As shown in the equation below, social planner's optimal condition contains an extra term, $\mathbb{E}_{t}\left[\widetilde{\mu}_{t+1} \Phi_{t+1}\left(1+\delta q_{t+1}^{L}-\frac{q_{t}^{L}}{q_{t}^{S}}\right)\right]$, in the insurance benefit of long-term debt. This term is precisely pecuniary externality.

$$
\begin{aligned}
\mathbb{E}_{t}\left[\widetilde{u}_{T t+1}^{\prime}\left(\frac{1}{q_{t}^{S}}-\frac{1+\delta q_{t+1}^{L}}{q_{t}^{L}}\right)\right]+\mathbb{E}_{t}\left[\widetilde{\mu}_{t+1} \Phi_{t+1}\left(\frac{1}{q_{t}^{S}}-\frac{1+\delta q_{t+1}^{L}}{q_{t}^{L}}\right)\right] & =0 \\
\mathbb{E}_{t}\left[\widetilde{u}_{T t+1}^{\prime} \cdot t p_{t}\right]+\mathbb{E}_{t}\left[\widetilde{\mu}_{t+1} \Phi_{t+1} \cdot t p_{t}\right] & =0
\end{aligned}
$$

What does the pecuniary externality term stand for in the insurance benefit of long-term debt? Fundamentally, insurance benefit of long-term debt stems from the fact that long-term debt's effectively repayment is smaller than that of short-term debt in bad states, bringing in resource to future adverse states, as confirmed by Figure 8. The pecuniary externality term, $\mathbb{E}_{t}\left[\widetilde{\mu}_{t+1} \Phi_{t+1}\left(1+\delta q_{t+1}^{L}-\frac{q_{t}^{L}}{q_{t}^{S}}\right)\right]$, captures difference between the undervaluation of repayment cost associated with short-term debt and long-term debt, and this relative cost undervaluation leads to inefficient maturity structure and calls for maturity-dependent capital control policies.

To shed light on the extent of maturity-dependence and how short- and long-term inflow controls should differ from each other, it is important to analyze the variation in the magnitude of the relative cost undervaluation between short-term debt and long-term debt along business cycle.

Basically, the pecuniary externality here represents the expected marginal utility from swapping to long-term borrowing through the collateral price and borrowing limit feedback loop. Therefore, the total pecuniary externality can be decomposed to two parts: one is the undervaluation of future repayment cost per unit of repayment, and the other is the amount of repayment decrease due to the insurance benefit of long-term debt. As shown in the cyclicality of capital control, the first component peaks in crisis due to the large probability of collateral constraint binding and the significant magnitude of financial amplification. The second component, which is unique to maturity structure choice, is determined by term premium. Therefore, the higher the term premium is, the larger the pecuniary externality, the greater relative repayment cost undervaluation of short-term liabilities by private agents.

Intuitively, high term premium means large drop in the future value of long-term liability, equivalently, large decrease in effective repayment by swapping to long-term borrowing. Ultimately, the spared resource is able to remarkably improve future bad states through collateral price and borrowing limit feedback loop, i.e. large pecuniary externality. As shown in Arellano and Ramanarayanan (2012), term premium is positive on average and decreases in crises, hence, the pecuniary externality resembles the dynamics. In terms of inefficiency of maturity choice, this means that there is more cost undervalue of short-term liabilities on average, and that the undervalue is most severe in crises. To counteract the inefficiency, capital control tightening should emphasize short-term inflow, especially in crises.

Figure 14: Optimal Long-term Capital Control Without Term Premium


Note: This plot presents the dynamics of short-/long-term debt taxes in financial crisis episode in a counterfactual case with term premium half as the original calibration.

To quantify the role of term premium in generating higher optimal short-term capital control tax than the long-term, I simulate the model with small term premium ${ }^{21}$ and plot the new optimal long-term capital control tax in Figure 14. With small term premium, the optimal long-term capital control tax is very similar to that on the short-term. Taking to the extreme, if term premium is negative instead of positive, applying the same logic, the pattern of short- and long-term capital controls will be overturned which will feature long-term inflow targeting instead of short-term.

Besides the role in the undervaluation of insurance benefit, another important function of term premium is to generate interior solution of optimal maturity structure. To see this, consider zero term premium case, which can be viewed as the debt price generated by a risk-neutral international creditor, i.e. $M_{t+1}=M$, term premium will be nil.

$$
\begin{aligned}
q_{t}^{S} & =\mathbb{E}_{t} M_{t+1}=M \\
q_{t}^{L} & =\mathbb{E}_{t}\left[M_{t+1}\left(1+\delta q_{t+1}^{L}\right)\right]=\mathbb{E}_{t}\left[M\left(1+\delta q_{t+1}^{L}\right)\right]=q_{t}^{S} \mathbb{E}_{t}\left(1+\delta q_{t+1}^{L}\right) \\
\Longrightarrow t p_{t} & =0
\end{aligned}
$$

Proposition 2. Without term premium, the competitive equilibrium yields a corner solution with borrowing up to bind collateral constraint in terms of long-term debt.

Recall that private agents in the small open economy is risk-averse, therefore, they value the fact that long-term debt provides insurance against averse shocks tomorrow. However, the risk-neutral investors do not assign positive value to this, hence, they are indifferent between whether the same amount of repayment is returned in good state or bad. As a result, the risk-neutral international investors take all the risk and provide insurance for the borrowers, resulting in borrowing exclusively in long-term debt contract. (See Appendix for detailed proof).

## E. Debt Portfolio In Competitive Equilibrium and Ramsey Equilibrium

As shown in Section 4.C, Ramsey equilibrium has fewer crises and less severe crises, and one important difference between the two is the external borrowing decision. In Ramsey equilibrium, total borrowing falls less than that in competitive equilibrium, and the share of short-term debt is much smaller. This section aims to provide more information on the comparison on the debt portfolio between the two types of equilibria, going beyond the crisis episodes.

[^16]Total Debt - Social planner and private agents differ in their calculation of the expected future repayment cost, and this difference will be factored into the optimal total borrowing and result in different debt volume. To analyze the choice of debt volume in competitive equilibrium and Ramsey equilibrium, I compare the total debt's ergodic distribution in the two equilibria. Since social planner internalizes the financial amplification effect of future repayment cost through the feedback loop of consumption decrease, collateral price fall, and borrowing capacity contract, social planner restrains from over consume today and takes a less indebted position than private agent.

To demonstrate the correction of over-borrowing by optimal capital controls, Figure 15 (left panel) plots the ergodic distribution of total debt accumulation in Ramsey economy and competitive economy, respectively. The pattern implies that the competitive economy assign higher probability to holding large amount of debt. The average total debt stock in competitive economy (0.89) is significantly higher than the Ramsey economy counterpart (0.79).


Note: This plot shows the ergodic distribution of total debt and short-term debt share, respectively. The competitive equilibrium is shown in solid line, and the Ramsey equilibrium is the dashed line.

Debt Maturity - Another important aspect of debt policy is the maturity choice. As discussed in Section 5.D, the optimal maturity structure is pinned down by the trade-off between the insurance benefit of long-term debt versus the cost benefit of short-term debt, and that the presence of pecuniary externality affects the calculation of long-term debt's insurance benefit. Although both
private agents and social planner acknowledge that issuing long-term debt today helps to transfer funds to tomorrow's bad states, social planner values the transfer even more by internalizing its influence on easing collateral constraint in tomorrow's bad states. Therefore, social planner selects a portfolio with longer maturity than private agent.

Figure 15 confirms that optimal portfolio in Ramsey economy tilts towards long-term debt than the short-term. On average, competitive equilibrium holds a portfolio constituted by $23.2 \%$ of short-term debt, while Ramsey equilibrium only assigns $13.2 \%$.

Figure 16: Welfare Gains from Optimal Capital Controls
Welfare improvement: $\omega$


Note: This chart plots the welfare gain of optimal capital controls measured by $\omega$. Each curve is evaluated at a fixed level of current long-term debt stock, averaged across exogenous states, and with varying current short-term debt holding.

## F. Welfare Analysis

Welfare Gain - Given that the optimal capital controls substantially correct over-borrowing and short-term debt overtaking, this part tends to quantify how much does the economy benefit from these regulations. To do so, I apply the same method of Bianchi (2011) to define the welfare gain as the proportional increase in consumption for all states in the competitive equilibrium that would make private agents indifferent between remaining in the competitive equilibrium and the Ramsey equilibrium. Formally, the welfare gain $\left\{\omega\left(d_{t}^{S}, d_{t}^{L}, \mathbf{S}_{t}\right)\right\}$ is calculated according to the following equation.

$$
\begin{equation*}
\left(1+\omega\left(d_{t}^{S}, d_{t}^{L}, \mathbf{S}_{t}\right)\right)^{1-\sigma} V\left(d_{t}^{S}, d_{t}^{L}, \mathbf{S}_{t}\right)=\widetilde{V}\left(d_{t}^{S}, d_{t}^{L}, \mathbf{S}_{t}\right) \tag{16}
\end{equation*}
$$

where $V(\cdot)$ is the value function in competitive equilibrium and $\tilde{V}(\cdot)$ is that in Ramsey equilibrium. The level of $\omega$ indicates the magnitude of increase in welfare generated by optimal capital controls.

Figure 16 presents the distribution of welfare gain with respect to current debt levels. First, for the majority cases, optimal capital control policies improves welfare as expected, no matter starting with low liability or high debt burden. Second, the magnitude of welfare improvement is positively correlated with the level of debt. Intuitively, with high level of debt, the economy is vulnerable to future adverse shocks and results in crisis. In these states, the optimal capital controls has the largest correction effect in constraining over-borrowing and preventing crisis. Thirdly, as has been shown that crises periods require higher capital control tax rates, a suboptimal constant tax rate must lie between the lower level in tranquil times and the higher level in crises times. As a result, the high tax rate may discourage consumption and borrowing in states with favorable borrowing conditions. For instance, in Figure 16, with low liabilities, meaning decent financial outstanding, welfare gain is slightly negative. All the states taken together, the optimal capital controls raise welfare equivalent to $0.59 \%$ increase in consumption on average.

The previous sections have established that the Ramsey equilibrium can be supported by a set of state-contingent and maturity-dependent capital controls in the competitive equilibrium, and that the optimal capital controls significantly improve welfare. Given the significant welfare enhancement of optimal policies, this section analyzes the welfare effect under sub-optimal policies, i.e. state-independent or maturity-independent. To do so, I consider the same environment as the benchmark but limit social planner to be able to use only fixed-rate capital control taxes. In particular, I consider four cases: i) fixed taxes but maturity-dependent ( $\tau_{t}^{S}=\tau^{S}, \tau_{t}^{L}=\tau^{L}$ ), ii) fixed tax and maturity-independent $\left(\tau_{t}^{S}=\tau_{t}^{L}=\tau\right)$, iii) only fixed tax on short-term inflow $\left(\tau_{t}^{S}=\tau^{S}, \tau_{t}^{L}=0\right)$, and iv) only fixed tax on long-term inflow $\left(\tau_{t}^{S}=0, \tau_{t}^{L}=\tau^{L}\right) .{ }^{22}$ In general, these cases tend to provide insights on policy implementations when regulations are challenging to implement in practice, in particular, the last two are also set to gauge the relative importance of short- and long-term inflow capital controls.

Without state-dependent capital control taxes, the decentralization of Ramsey equilibrium is no longer feasible. Consequently, social planner now solves for the optimal level of fixed capital control tax to maximize household's expected life-time utility with the post-tax borrowing rates. Since the state-contingent optimal capital controls is procyclical, if set equally, social planner trades off between the benefit of high tax rate in preventing crisis and the corresponding cost in depressing

[^17]consumption in normal time.
Definition 5. (Constrained Social Planner's Optimization Problem) Social planner chooses $\tau$ to maximize the value function resulted from the competitive equilibrium accordingly to Definition 1 under the post-tax effective borrowing rates.

Table 5 presents the welfare effect of different types of capital controls compared to the unregulated competitive benchmark. The first observation is that the optimal capital control tax, even set fixed, is able to insure the economy against adverse shocks, reduce crisis frequency, and improve welfare on average. For instance, the maturity-dependent fixed taxes are able to achieve $59 \%$ of the welfare improvement under optimal capital controls, and even the least effective one, fixed tax on long-term inflow, generates positive welfare improvement. This result is reasonable given that in the model, pecuniary externality calls for positive tax rate even in normal time. Second, state-contingency contributes a significant part of welfare improvement. From the optimal case to state-independent case i), the welfare gain drops by $40 \%$. However, from the maturity-dependent case i) to maturity-independent case ii), the welfare gain drops by a smaller extent, $13 \%$.

Regarding the individual effectiveness of short-term inflow control and long-term inflow control, if restricted with one type of fixed tax, the economy will benefit more from a fixed short-term capital control tax. The flat short-term capital control tax is able to achieve about $32 \%$ the welfare improvement in optimal state-contingent and maturity-dependent regulations, while the welfare gain under optimal long-term fixed tax is about $3.4 \%$ of Ramsey. It is intuitive to see short-term capital control is superior to the long-term because the overborrowing in competitive equilibrium is more severe in short-term debt than the long-term. In other words, given the amount of over-borrowing, short-term borrowing should be more tightly regulated.

Table 5: Welfare Effect of Constrained Optimal Capital Controls

|  | Laissez-faire | Fixed iv) | Fixed iii) | Fixed ii) | Fixed i) | Optimal |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\tau_{t}^{S}=0$ | $\tau_{t}^{S}=0$ | $\tau_{t}^{S}=\tau^{S}$ | $\tau_{t}^{S}=\tau$ | $\tau_{t}^{S}=\tau^{S}$ | $\tau_{t}^{S *}$ |
|  | $\tau_{t}^{L}=0$ | $\tau_{t}^{L}=\tau^{L}$ | $\tau_{t}^{L}=0$ | $\tau_{t}^{L}=\tau$ | $\tau_{t}^{L}=\tau^{L}$ | $\tau_{t}^{L *}$ |
| crisis frequency | 11.8 | 13.3 | 15.3 | 16.7 | 19.2 | 26.3 |
| welfare gain | - | $0.02 \%$ | $0.19 \%$ | $0.23 \%$ | $0.35 \%$ | $0.59 \%$ |
| $\tau^{S}$ | - | - | $1.83 \%$ | $1.52 \%$ | $2.09 \%$ | $2.79 \%$ |
| $\tau^{L}$ | - | $0.65 \%$ | - | $1.52 \%$ | $0.98 \%$ | $1.71 \%$ |

Note: This table reports the welfare improvement by different types of capital control taxes. Crisis frequency is measured in year, and welfare gain is defined as $\omega$ in (16) and the mean of $\omega$ is reported.

## 6 Conclusion

This paper studies debt portfolio in external borrowing and optimal capital control policies with financial friction. Empirically, I construct a new measure on changes in capital control policies and document two novel stylized facts about capital inflow controls in financial crises: tightening in crises and short-term inflow targeting. To rationalize the empirical patterns, this paper builds a small open economy model with multiple maturity debt, collateral constraint, and risk-averse international creditors. The model highlights the impact pecuniary externality introduced by collateral constraint in generating inefficient debt portfolio in competitive equilibrium and derives a set of state-contingent and maturity-dependent capital controls for reattaining social optimal debt portfolio.

In the model, private agents balance between the benefit of borrowing and the cost of debt repayment to determine optimal external borrowing. In the meantime, they select the optimal maturity structure by considering the trade-off between the insurance benefit of long-term debt in hedging against the future adverse shocks and the cost benefit of issuing short-term debt. However, with the presence of collateral constraint, private agents fail to internalize the feedback loop between collateral price and borrowing limit and undervalue the repayment cost of financial liabilities. Specifically, the cost undervaluation is especially severe for short-term liabilities due to its high repayment relative to long-term debt in constrained states. As a result, the competitive equilibrium exhibits overborrowing and excess short-term debt compared to the social optimal. To fully offset the inefficiencies, capital controls on short- and long-term inflows should tighten in crisis due to the high probability of continuing crisis, and short-term inflow should be tightened by a larger extent due to the positive term premium, especially in crisis. Implemented optimally, capital controls can substantially improve welfare, reducing the crisis frequency by half and alleviating the crisis severity.

Overall, the paper's main innovation has been to study the effect of financial friction on maturity choice and the optimal policy in correcting the pecuniary externality. There are other dimensions in which the model can be extended. First, the model now concentrates on external borrowing and its corresponding optimal regulating policy. However, as shown by Broner et al. (2013), large inflow tends to be accompanied by large outflow, and capital inflow and outflow tightening is highly likely to be implemented simultaneously (Fernández, Rebucci and Uribe (2015)). Therefore, it would be of great interest to incorporate capital outflow decisions in the small open economy and to analyze its interaction with financial frictions. Second, in deriving optimal capital control policies, I assume that social planner has the ability to commit to policy. This might change under different forms of
collateral such as asset. To examine the Markov perfect equilibrium with external borrowing at different maturities would be an important complement to the study in this paper.

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## Appendix

## A. Procedure for Constructing Index of Changes in Capital Control

Based on AREAER, the construction of changes in capital controls takes the following steps.

1. Extract the narratives of policies regarding capital transaction in the "Changes in the previous year" at the end of country profile from AREAER.

Post-1998 years are directly downloaded from AREAER website. From 1983 to 1998 is extracted manually from each AREAER. Prior to 1983, the policies are listed in the chronological manner without indicating whether it is about capital transaction or not. Therefore, discretion is used for the selection.
2. Extract the maturity-related policies.

First, search for the policies whose narratives contain "day", "week", "month", "quarter", "year", "short", and "long". Second, label whether it targets short-term (intending to lengthening the average maturity) or long-term (intending to shortening the average maturity).
3. Each policy is characterized by three labels: the direction of transaction, whether it is a tightening or easing, whether it is associated with short-term, or long-term, or independent.

## B. Proof of Proposition 1

The regulated competitive equilibrium is defined as

$$
\begin{align*}
\max _{c_{t}^{T}, c_{t}^{N}, d_{t+1}^{S}, d_{t+1}^{L}, f_{t}, \tau_{t}^{S}, \tau_{t}^{L}} & \mathbb{E}_{0} \sum_{0}^{\infty} \beta^{t} U\left(c_{t}^{T}, c_{t}^{N}\right) \\
\text { s.t. } U_{T t}^{\prime} & =\lambda_{t}  \tag{A.1}\\
p_{t} & =\frac{1-\alpha}{\alpha}\left(\frac{c_{t}^{T}}{c_{t}^{N}}\right)^{1 / \xi}  \tag{A.2}\\
\lambda_{t}\left[\left(1-\tau_{t}^{S}\right) q_{t}^{S}-\mu_{t}\right] & =\beta \mathbb{E}_{t} \lambda_{t+1}  \tag{A.3}\\
\lambda_{t}\left[\left(1-\tau_{t}^{L}\right) q_{t}^{L}-\mu_{t}\right] & =\beta \mathbb{E}_{t}\left[\lambda_{t+1}\left(1+\delta\left(1-\tau_{t+1}^{L}\right) q_{t+1}^{L}\right)\right]  \tag{A.4}\\
\mu_{t} & \geq 0  \tag{A.5}\\
\mu_{t}\left[\kappa\left((1-\gamma) f_{t}^{\gamma}+p_{t} y_{t}^{N}\right)-d_{t+1}^{S}-d_{t+1}^{L}\right] & =0  \tag{A.6}\\
d_{t+1}^{S}+d_{t+1}^{L} & \leq \kappa\left[\Gamma(1-\gamma) f_{t}^{\gamma}+p_{t} y_{t}^{N}\right]  \tag{A.7}\\
c_{t}^{N} & =y_{t}^{N}  \tag{A.8}\\
c_{t}^{T}+d_{t}^{S}+d_{t}^{L} & =\Gamma(1-\gamma) f_{t}^{\gamma}+q_{t}^{S} d_{t+1}^{S}+q_{t}^{L}\left(d_{t+1}^{L}-\delta d_{t}^{L}\right)  \tag{A.9}\\
\Gamma \gamma f_{t}^{\gamma-1} & =1+\left(\frac{1}{q_{t}^{S}}-1\right) \eta \tag{A.10}
\end{align*}
$$

Lemma 1. Any process $\left\{c_{t}^{T}, d_{t+1}^{S}, d_{t+1}^{L}\right\}$ satisfy (A.1) to (A.10) if and only if they satisfy (A.9), (A.10), and

$$
\begin{equation*}
d_{t+1}^{S}+d_{t+1}^{L} \leq \kappa\left[\Gamma(1-\gamma) f_{t}^{\gamma}+\frac{1-\alpha}{\alpha}\left(\frac{c_{t}^{T}}{c_{t}^{N}}\right)^{1 / \xi} y_{t}^{N}\right] \tag{A.11}
\end{equation*}
$$

Proof. From (A.9) - (A.11) to (A.1) - (A.10):
From $c_{t}^{T}, c_{t}^{N}$, we can define $\lambda_{t}$ using (A.1) and $p_{t}$ using (A.2). Then, we set $\mu=0$ which validates (A.5) and (A.6), and we choose $\tau_{t}^{S}, \tau_{t}^{L}$ according to (A.3) and (A.4). With $p_{t}$ satisfying (A.2), (A.7) is equivalent to (A.11).

From (A.1) - (A.10) to (A.9) - (A.11): plugging (A.2) to (A.7) yields (A.11).
Lemma 1 suggests that the regulated competitive equilibrium is equivalent to Ramsey equilibrium with properly defined taxes, which essentially proves that Ramsey equilibrium can be decentralized.

## C. Proof of Proposition 2

Proof. Suppose that the collateral constraint does not bind, then the corresponding Euler equations for short- and long-term debt are

$$
\begin{aligned}
q_{t}^{S} & =\mathbb{E}_{t} \frac{\beta u_{t+1}^{\prime}}{U_{t}^{\prime}} \\
q_{t}^{L} & =\mathbb{E}\left[\frac{\beta u_{t+1}^{\prime}}{u_{t}^{\prime}}\left(1+\delta q_{t+1}^{L}\right)\right]=\mathbb{E}_{t} \frac{\beta u_{t+1}^{\prime}}{u_{t}^{\prime}} \mathbb{E}_{t}\left(1+\delta q_{t+1}^{L}\right)+\delta \operatorname{Cov}\left(\frac{\beta U_{t+1}^{\prime}}{U_{t}^{\prime}}, q_{t+1}^{L}\right) \\
& =q_{t}^{S} \mathbb{E}_{t}\left(1+\delta q_{t+1}^{L}\right)+\delta \operatorname{Cov}\left(\frac{\beta u_{t+1}^{\prime}}{u_{t}^{\prime}}, q_{t+1}^{L}\right)
\end{aligned}
$$

With risk-neutral foreign investor, the long-term debt's price is associated with the price of the short-term.

$$
q_{t}^{L}=q_{t}^{S} \mathbb{E}_{t}\left(1+\delta q_{t+1}^{L}\right)
$$

Therefore, if the covariance term is positive ${ }^{23}$, there will not be long-term debt, otherwise, there will not be short-term debt.

## D. Ruling Out Multiple Equilibria

Schmitt-Grohe and Uribe (2016) show that under plausible calibrations, there exist multiple equilibria with both underborrowing and overborrowing. Multiple equilibria stem from a self-fullfiling decline in nontradable goods price leading to reduction in collateral value, consumption, and nontradable goods price in a compatible feedback loop. Specifically, when collateral constraint binds, tradable goods consumption is pinned down by the collateral value which hinges on the level of consumption itself.

$$
\begin{equation*}
c_{t}^{T}=\kappa\left[y_{t}^{T}+y_{t}^{N} \frac{1-\alpha}{\alpha}\left(\frac{c_{t}^{T}}{c_{t}^{N}}\right)^{1 / \xi}\right] \equiv f\left(c_{t}^{T}\right) \tag{A.12}
\end{equation*}
$$

As discussed in Jeanne and Korinek (2012) and Benigno et al. (2016), when the elasticity of substitution between tradable and nontradable goods is less than one, a sufficient condition of unique equilibrium requires $\lim _{c_{t}^{T} \rightarrow 0} f^{\prime}\left(c_{t}^{T}\right)=0, \lim _{c_{t}^{T} \rightarrow \infty} f^{\prime}\left(c_{t}^{T}\right)=\infty$, and $\left.f^{\prime}\left(c_{t}^{T}\right)\right|_{c_{t}^{T}=f\left(c_{t}^{T}\right)}>1$. These conditions are satisfied in the calibration.

Intuitively, given tradable and nontradable goods are complements, when extra borrowing increases tradable consumption, nontradable consumption will rise simultaneously. As a result, nontradable price goes up, so will collateral value and borrowing limit. If they are substitutes, then

[^18]borrowing increase will lead to nontradable consumption decrease and reduce borrowing limit.
Another multiple equilibria issue that might arise in the model is that the debt portfolio could be undetermined when collateral constraint binds. This can be ruled out if for each portfolio supporting $c_{t}^{T}$ under binding collateral constraint there exists a unique level of short-term debt (or long-term debt) given the level of long-term debt (short-term debt). The condition is satisfied in the model because the total debt value is a linear combination of short- and long-term debt given their prices.

Figure A.1: External Borrowing


Figure A.2: Share of Short-term External Debt


Figure A.3: Dynamics and Optimal Capital Controls in Boom-Bust Periods








Note: This chart plots the dynamics of key aggregate vanisbles and optimal capital controls in boom-bust periods.

Figure A.4: Dynamics and Optimal Capital Controls in Crises Periods


Note: This chart plots the dynamics of key aggregate variables and optimal capital controls in financial crises periods under both SDF shock and $y^{N}$ shock.


[^0]:    *International Monetary Fund, email: jzhou@imf.org, website: http://jingzhoujz.weebly.com/. I thank Martín Uribe, Stephanie Schmitt-Grohé, and Shang-Jin Wei, for their advice and support throughout the project. I also thank JaeBin Ahn, Yan Bai, Carlos Carvalho, YiLi Chien, Keshav Dogra, Jonathan Eaton, Miguel Faria-e-Castro, Juan Hatchondo, Gee Hee Hong, Sungki Hong, Rui Mano, B. Ravikumar, Juan Sánchez, Jesse Schreger, Daniel Villar, Yi Wen, Pierre Yared, and seminar participants at Columbia University for their comments. This paper was partly written at the Federal Reserve Bank of St. Louis, while I was a dissertation fellow. I am grateful for their hospitality and the financial support from AEA summer fellowship. The views expressed in this paper do not necessarily represent the views of the IMF, its Executive Board, or IMF management.

[^1]:    ${ }^{1}$ For the main stylized facts see for instance, Gourinchas, Valdes and Landerretche (2001), Mendoza and Terrones (2008), Rogoff and Reinhart (2009), Broner et al. (2013).
    ${ }^{2}$ See Brunnermeier (2009), Arellano and Ramanarayanan (2012), Broner, Lorenzoni and Schmukler (2013).
    ${ }^{3}$ See Ostry et al. (2010), Rey (2015), Korinek (2017), Jeanne and Korinek (2010), and Bianchi (2011), among many others.
    ${ }^{4}$ See, for example, Cole and Kehoe (2000), Alfaro and Kanczuk (2009), Jeanne (2009).

[^2]:    ${ }^{5}$ The "Annual Report on Exchange Arrangements and Exchange Restrictions" published by IMF provides a comprehensive profile for each country's capital transaction regulations, details in Section 3.

[^3]:    ${ }^{6}$ One important exception is Korinek (2017), which discusses the general principle of capital control tax with respect to different types of capital flows. However, it does not endogenize the maturity structure of external borrowing and its interaction with capital control policy.

[^4]:    ${ }^{7}$ Similar procedures are conducted for capital outflow controls.

[^5]:    ${ }^{8}$ Detailed procedure can be found in Appendix.

[^6]:    ${ }^{9}$ Capital inflow could take the form of domestic currency deposit as well, and likewise foreign currency deposit could be from domestic fund. Therefore, I check the domestic currency term deposit and demand deposit reserve requirement for the same countries and same period as foreign currency, and the domestic currency reserve requirement with respect to different maturity shows a qualitatively similar pattern with the foreign counterpart.

[^7]:    ${ }^{10}$ Following Jeanne and Korinek (2010), the collateral constraint can be micro-founded in the same way as the moral hazard problem. Assume that international creditors cannot coordinate to punish the borrower by excluding him from borrowing in future periods. In the meantime, the borrower has the option to invest in a scam that allows him to remove his future endowment income from the reach of his current creditors. This would allow him to default on his debts next period without facing a penalty. However, the creditors can observe the scam in the current period and take the insider to court before the scam is completed. If they do so, they can seize a fraction $\kappa$ of the borrower's collateral, where $\kappa$ captures imperfect legal enforcement. The creditor can re-sell the collateral at the prevailing market price. This implies the participation condition for the creditor to lend to the borrower should take the form of (3).

[^8]:    ${ }^{11}$ Here the timing is that the loan is made at the beginning of each period and repaid at the end of the period, so the contemporaneous debt price is used, same as Mendoza (2010). However, Neumeyer and Perri (2005) Chang and Fernández (2013) use the last period interest rate.
    ${ }^{12}$ The link between borrower's income and borrowing rate has been convincingly justified in the default literature as income affects the probability of default and hence the price of borrowing. However, without default, it is less straightforward to associate the external borrowing rate to borrower's income other than the collateral constraint channel embedded in the model. Here I take working capital constraint framework, which is a workhorse model used in analyzing the business cycle of emerging countries in response to external interest rate, for instance, Neumeyer and Perri (2005), Uribe and Yue (2006), etc. Alternative justification could be both borrower's tradable income and lender's risk appetite depend on world-wide income shock.
    ${ }^{13}$ This specification of the lender's stochastic discount factor is a special case of the one-factor model of the term structure, and it has been used in models of sovereign default, for instance, Arellano and Ramanarayanan (2012), Bocola and Dovis (2016).

[^9]:    ${ }^{14}$ Note that $\widetilde{\mu}_{t}=\frac{1}{1-\Phi_{t}} \cdot \mu_{t}$, because social planner internalizes the financial amplification effect of extra collateral through the feedback loop of consumption increase, collateral value rise, and borrowing limit relax, when evaluating the shadow price of collateral. This is the same result as Korinek (2017).

[^10]:    15 "Effective" repayment of long-term debt is the discounted present value of all the future payments of long-term debt. Alternatively, it could be viewed as the cost of retiring one unit of long-term debt.

[^11]:    ${ }^{16}$ According to Benigno et al. (2016) and Schmitt-Grohé and Uribe (2016), there might exist multiple equilibria in the set of collateral constraint models. I discuss the current calibration guarantees unique equilibrium in Appendix.

[^12]:    ${ }^{17}$ Term structural data is from Broner, Lorenzoni and Schmukler (2013). 3-year spread is the shortest in their data, and 12 -year is the closest to the equivalent duration of 10 -year as calibrated in the data.

[^13]:    ${ }^{18}$ This analysis has been used in Aguiar and Amador (2013) and Bianchi, Hatchondo and Martinez (2012) to illustrate maturity choice in defautable debt.

[^14]:    ${ }^{19}$ Effective repayment of long-term debt is the discounted value of all the future repayments, and equivalently, it is also the cost of retiring the existing long-term debt with the new market debt price.

[^15]:    ${ }^{20}$ See Neumeyer and Perri (2005) and Uribe and Yue (2006) among many others.

[^16]:    ${ }^{21}$ To generate counterfactual small term premium, I keep the short-term rate unchanged and reduce the term premium by half.

[^17]:    ${ }^{22}$ Ideally, another case with state-contingent but maturity-independent taxes should be considered, however, the solution of constrained optimal taxes is difficult given the large number of states.

[^18]:    ${ }^{23}$ In the case without interest rate shock, the covariance term is zero, so the maturity structure is undetermined.

